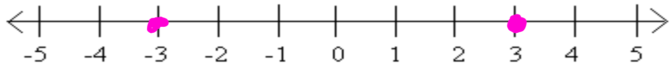
Unit 2: Solving Equations and Inequalities
Lesson 6: Solving Absolute Value Equations and Compound Inequalities

RECALL: Absolute value is the distance from zero. Therefore $|x| = 3$ asks which x-values are 3 units from 0?

$x=3$ $-x=3$



$x=3$
$x=-3$

Each absolute value can have up to two different solutions.

When **evaluating** expressions, absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

Example: Evaluate $|2x-3|+4$ if x=3

$|2(3)-3|+4$
$=|6-3|+4$
$=|3|+4 = 3+4 = \boxed{7}$

Evaluate $|2x-3|+4$ if x= -2

$|2(-2)-3|+4$
$=|-4-3|+4$
$=|-7|+4 = 7+4 = \boxed{11}$

Since absolute value equations always have a positive and negative answer. We must consider both sides when solving absolute values.

Example:

$$|x-4| = 7$$

Case 1	Case 2
$x-4=7$	$x-4=-7$
$x-4+4=7+4$	$x-4+4=-7+4$
$x=11$	$x=3$

Remember: Check your solutions.

Practice: Solve Algebraically:

a) $|x-13| = 4$

$x-13=4$
$+13\ \ +13$
$\boxed{x=17}$

$-(x-13)=4$
$-x+13=4$
$-13\ \ -13$
$\frac{-x}{-1} = \frac{-9}{-1}$ $\boxed{x=9}$

b) $|a-5|+6=7$
$-6\ \ -6$
$|a-5|=1$

$a-5=1$
$+5\ \ +5$
$\boxed{a=6}$

$-(a-5)=1$
$-a+5=1$
$-5\ \ -5$
$\frac{-a}{-1}=\frac{-4}{-1}$ $\boxed{a=4}$

What is the solution of $|x| = -4$

No solution because absolute value cannot be negative.
You can't go a negative distance.

c) $|3x-6|+3=2$
$-3\ \ -3$
$|3x-6|=-1$
No solution

d) $|3x+2| = 11$

$3x+2=11$
$-2\ \ -2$
$\frac{3x}{3}=\frac{9}{3}$
$\boxed{x=3}$

$-(3x+2)=11$
$-3x-2=11$
$+2\ \ +2$
$\frac{-3x}{-3}=\frac{13}{-3}$
$\boxed{x=\frac{-13}{3}}$

e) $|x+6| = 3x-2$

$x+6=3x-2$
$-3x\ \ -3x$
$-2x+6=-2$
$-6\ \ -6$
$\frac{-2x}{-2}=\frac{-8}{-2}$
$\boxed{x=4}$

$-(x+6)=3x-2$
$-x-6=3x-2$
$+x\ \ +x$
$-6=4x-2$
$+2\ \ +2$
$\frac{-4}{4}=\frac{4x}{4}$ $\cancel{x=-1}$

The absolute value of an equation gives the distance from a given center point.

Application:

Most meat thermometers are accurate to within plus or minus 2°F.

a. If a meat thermometer reads 150°F, write an equation to determine the least and greatest possible temperatures of the meat.

b. Solve your equation.

c. Ham needs to reach an internal temperature of 150°F to be fully cooked. To what temperature reading should you cook a ham to ensure that the minimum temperature is reached? Explain.

Solving Compound Inequalities

<u>Vocabulary</u>

Compound inequality:

Union:

Intersection:

- A number is a solution to an **AND** inequality when the number is a solution to **BOTH** inequalities. This is where the two solutions overlap.
- A number is a solution to a **OR** inequality if the number is a solution to **EITHER** inequality.

Ways to write a compound inequality

AND	OR
$5 < x \quad 3 < 10$	$2x + 4 \quad 7$ or $x < \quad 10$
$x \quad 3 < 10$ and $x \quad 3 > 5$	

Let's Solve a Compound Inequality

Solve: $13 < 2x + 7 \leq 17$

Method 1: Solve both parts at the same time

$$13 < 2x + 7 \leq 17$$
$$-7 \quad -7 \quad -7$$
$$\frac{6}{2} < \frac{2x}{2} \leq \frac{10}{2}$$
$$\boxed{3 < x \leq 5}$$

Method 2: Solve each inequality separately

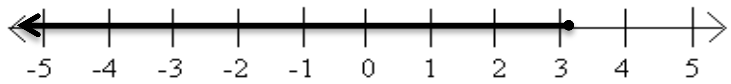
$$2x + 7 \leq 17$$
$$-7 \quad -7$$
$$\frac{\cancel{2}x}{\cancel{2}} \leq \frac{10}{2}$$
$$x \leq 5$$

$$2x + 7 > 13$$
$$-7 \quad -7$$
$$\frac{\cancel{2}x}{\cancel{2}} > \frac{6}{2}$$
$$x > 3$$

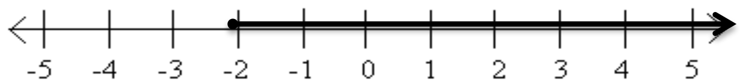
$$\boxed{x \leq 5 \text{ and } x > 3}$$

To check your answers, Graph the solution for each inequality on the number line and find it's intersection.

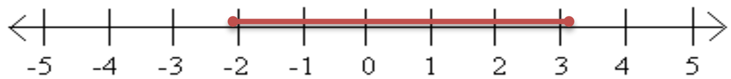
$x \quad 3$



$x > \quad 2$



$2 < x \quad 3$



What does this solution mean?

PRACTICE: Solve these compound inequalities.

a. $p + 5 < 8$ ***or*** $p - 3 > 1$

$\frac{-5 \; -5}{p < 3}$ ↓ $\frac{+3 \; +3}{p > 4}$

$\boxed{p < 3 \text{ or } p > 4}$

b. $-4 \leq 3x - 1 < 14$

$\frac{+1 \quad +1 \quad +1}{-3 \leq 3x < 15}$

$\frac{-3}{3} \leq \frac{3x}{3} < \frac{15}{3}$

$\boxed{-1 \leq x < 5}$

c. $\frac{3}{4}x - 2 < 1$ ***and*** $\frac{1}{2}x - 4 \geq -2$

$\frac{+2 \quad +2}{\cancel{4} \cdot \frac{3}{\cancel{4}}x < 3 \cdot 4}$

$\frac{\cancel{3}x}{\cancel{3}} < \frac{12}{3}$

$x < 4$

$\frac{+4 \quad +4}{\cancel{2} \cdot \frac{1}{\cancel{2}}x \geq 2 \cdot 2}$

$x \geq 4$

$\boxed{x < 4 \text{ and } x \geq 4}$

Application:

Perri is building a fence around a rectangular plot. The space allows for a perimeter of 17-20 yards. The width of the plot is 5 yards.

a. Write a compound inequality to describe the range of possible lengths of the plot.

b. Solve your inequality.