NOTES: ALGEBRA 2

SOLVING SYSTEMS ALGEBRAICALLY PART 1 (SUBSTITUTION)

STARTER:

1. Solve the system by graphing $\begin{aligned} x - 3y &= 9 \\ x + 2y &= -1 \end{aligned}$ and range.

$\frac{-3y}{-3} = \frac{-x+9}{-3}$

$y = \frac{1}{3}x - 3$

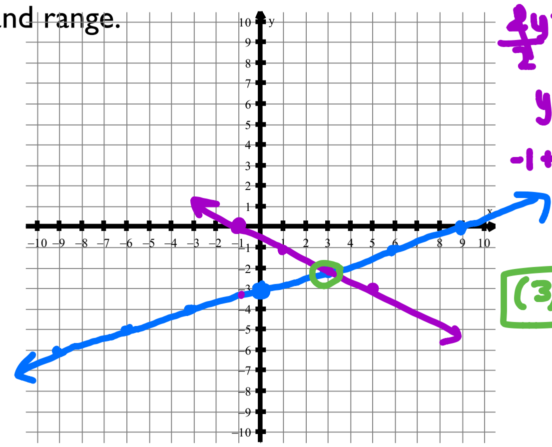
$\frac{2y}{2} = \frac{-x-1}{2}$

$y = -\frac{1}{2}x - \frac{1}{2}$

$-1 + 2y = -1$

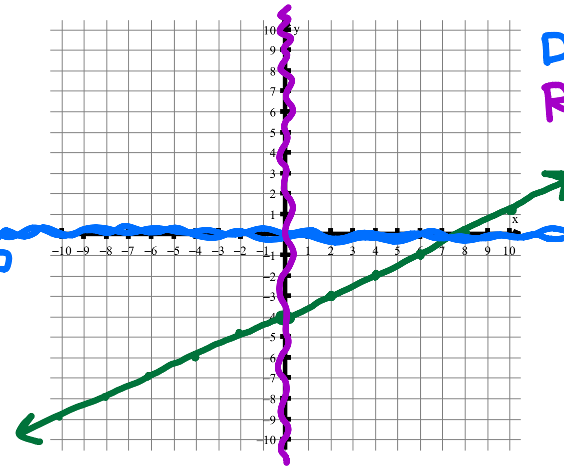
$2y = 0$

$y = 0$



$\boxed{(3,-2)}$

2. Graph $y = \frac{1}{2}x - 4$ and state the domain



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$-\infty$ ∞

3. Solve $\frac{\cancel{2}|x+3|}{\cancel{2}} = \frac{10}{2}$

$|x+3| = 5$

$x + 3 = 5$
$-3 \quad -3$
$\boxed{x = 2}$

$-(x+3) = 5$
$-x - 3 = 5$
$+3 \quad +3$
$-x = 8$

$\boxed{x = -8}$

4. Solve $\frac{-\cancel{3}|x-2|}{-\cancel{3}} = \frac{6}{-3}$

$|x-2| = -2$

$\boxed{\text{No solution}}$

Example 1: Given $y = 4x - 2$ evaluate when $x = 3$.

$y = 4(3) - 2$
$= 12 - 2$
$= 10$

$\boxed{y = 10}$

Sometimes, solving systems of equations by graphing can be difficult to determine the exact coordinates of the point of intersection from the graph. For systems of equations where it is hard to determine the point of intersection, it may be easier to solve the system algebraically.

There are two methods we use to solve a system algebraically: SUBSTITUTION and ELIMINATION.

SUBSTITUTION METHOD

When you think of substitution, think of *replacing*. Just like the example above, you replaced "x" with what you knew "x" was equal to.

STEPS:

1. Solve one equation for one variable. ($y = mx + b$ or $x = ay + c$ where *m, b, a, c* are all constants)
2. Substitute the expression for the variable in the other equation and solve.
3. Don't forget to substitute back into an original expression to find the second variable.

***Solutions should be written as (x,y)**

Example 2: Use substitution to solve the system of equations.

a. $3x + y = 7$
$4x + 2y = 16$

$3x+y=7$
$-3x \quad -3x$
$y=-3x+7$

$4x+2(-3x+7)=16$
$4x-6x+14=16$
$-14 \quad -14$
$4x-6x=2$
$-2x=2$
$\frac{-2x}{-2}=\frac{2}{-2}$
$x=-1$

$4(-1)+2y=16$
$-4+2y=16$
$+4 \quad +4$
$\frac{2y}{2}=\frac{20}{2}$
$y=10$

$\boxed{(-1, 10)}$

b. $x - 3y = 9$
$x + 2y = -1$

$x-3y=9$
$+3y \quad +3y$
$x=3y+9$

$(3y+9)+2y=-1$
$5y+9=-1$
$-9 \quad -9$
$\frac{5y}{5}=\frac{-10}{5}$
$y=-2$

$x+2(-2)=-1$
$x-4=-1$
$+4 \quad +4$
$x=3$

$\boxed{(3,-2)}$

c. $g + 3h = 8$
$\frac{1}{3}g + h = 9$

$g+3h=8$
$-3h \quad -3h$
$g=8-3h$

$\frac{1}{3}(-3h+8)+h=9$
$-h+\frac{8}{3}+h=9$
$\frac{8}{3}=9$

$\boxed{\text{NO solution}}$

d. $2a - 4b = 6$
$-a + 2b = -3$

$-a+2b=-3$
$-2b \quad -2b$
$\frac{-a}{-1}=\frac{-2b-3}{-1}$
$a=2b+3$

$2(2b+3)-4b=6$
$4b+6-4b=6$
$6=6$

$\boxed{\text{infinite solutions}}$

e. $-5x + 4y = -17$
$5x + y = 2$

$5x+y=2$
$-5x \quad -5x$
$y=-5x+2$

$-5x+4(-5x+2)=-17$
$-5x-20x+8=-17$
$-25x+8=-17$
$-8 \quad -8$
$\frac{-25x}{-25}=\frac{-25}{-25}$
$x=1$

$5(1)+y=2$
$5+y=2$
$-5 \quad -5$
$y=-3$

$\boxed{(1,-3)}$

f. $x - 7y = 24$
$-4x + 3y = -21$

$x-7y=24$
$x=7y+24$

$-4(7y+24)+3y=-21$
$-28y-96+3y=-21$
$-25y-96=-21$
$+96 \quad +96$
$\frac{-25y}{-25}=\frac{75}{-25}$
$y=-3$

$x-7(-3)=24$
$x+21=24$
$-21 \quad -21$
$x=3$

$\boxed{(3,-3)}$

g. $3x + y = 3$
$-3x - y = 6$

$3x+y=3$
$-3x \quad -3x$
$y=-3x+3$

$-3x-(-3x+3)=6$
$-3x+3x-3=6$
$-3=6$

$\boxed{\text{NO Solution}}$

h. $-10x - 2y = -34$
$5x + y = 17$

$5x+y=17$
$-5x \quad -5x$
$y=-5x+17$

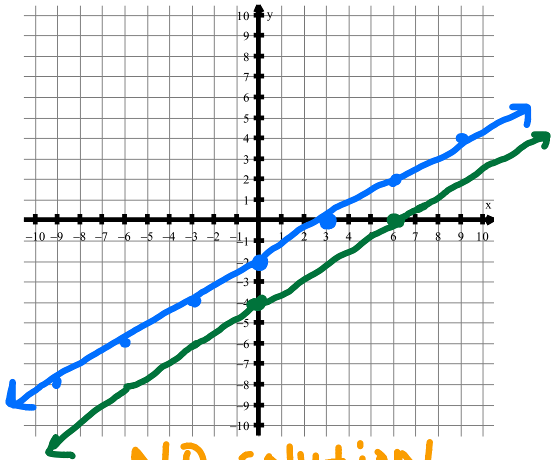
$-10x-2(-5x+17)=-34$
$-10x+10x-34=-34$
$-34=-34$

$\boxed{\text{Infinite solutions}}$

NOTES: ALGEBRA 2
SOLVING SYSTEMS ALGEBRAICALLY PART 2 (ELIMINATION)

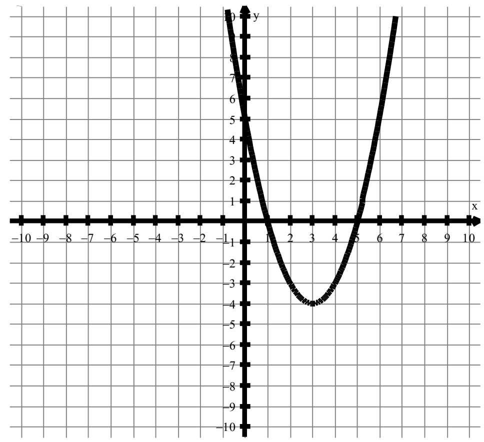
STARTER:

1. Solve the system by graphing $\begin{array}{l} 2x-3y=6 \\ 2x-3y=12 \end{array}$



NO solution

2. State the domain and range of the function



D: $(-\infty,\infty)$

R: $[-4,\infty)$

3. Simplify $(x-5)+(-2x^2+x)$

$\boxed{-2x^2+2x-5}$

4. Simplify $(-4a+3)-(a^2-a)$ $\quad -4a+(+a)$

$\boxed{-a^2-3a+3}$

ELIMINATION METHOD

When you think of elimination, think of *getting rid of*. We want to eliminate one of the variables by adding or subtracting the equations. When you add two true equations, the result is a new equation that is also true.

STEPS:

1. Determine what variable you want to eliminate. Make the coefficients of this variable opposites (the same number with opposite signs).
2. Add the two equations together and solve.
3. Don't forget to substitute back into an original expression to find the second variable.

***Solutions should be written as (x,y)**

Example 1: Use the elimination method to solve the system of equations.

a. $2x+4y=20$
$-2(x+y=6)$

$2x+4y=20$
$-2x-2y=-12$
$0x+2y=8$
$\frac{\cancel{2}y}{\cancel{2}}=\frac{8}{2}$
$y=4$

$x+(4)=6$
$-4 \quad -4$
$x=2$

$\boxed{(2,4)}$

b. $5b=20+2a$
$2a+4b=7$

$-2a+5b=20$
$2a+4b=7$
$0a+9b=27$
$\frac{\cancel{9}b}{9}=\frac{27}{9}$
$b=3$

$2a+4(3)=7$
$2a+12=7$
$-12 \quad -12$
$\frac{\cancel{2}a}{\cancel{2}}=\frac{-5}{2}$
$a=\frac{-5}{2}$

$\boxed{(\frac{-5}{2},3)}$

c. $5(2x+3y=12)$
$-2(5x-2y=11)$

$10x+15y=60$
$-10x+4y=-22$
$19y=38$
$\frac{19y}{19}=\frac{38}{19}$
$y=2$

$5x-2(2)=11$
$5x-4=11$
$+4 \quad +4$
$5x=15$
$\frac{\cancel{5}x}{5}=\frac{15}{5}$
$x=3$

$\boxed{(3,2)}$

d. $2(-3x+5y=12)$
$6x-10y=-21$

$-6x+10y=24$
$6x-10y=-21$
$0x+0y=3$
$0=3$

$\boxed{\text{NO solution}}$

e. $-3x+9y=9$
$9x-6y=-27$

f. $3(-5x-10y=0)$
$15x+30y=0$

$-15x-30y=0$
$15x+30y=0$
$0x+0y=0$
$0=0$

$\boxed{\text{infinite solutions}}$

g. $9x-20y=-15$
$-5x+10y=5$

h. $48x-8y=-16$
$6x-y=-7$

NOTES: ALGEBRA 2

SOLVING SYSTEMS FOR WORD PROBLEMS

STARTER:

1. Solve using substitution $\begin{cases} -6x-3y=-3 \\ x+7y=20 \end{cases}$

$\boxed{(-1,3)}$

$-7y \quad -7y$

$x=-7y+20$

$-6(-7y+20)-3y=-3$

$42y-120-3y=-3$

$39y-120=-3$

$+120 \quad +120$

$39y=117$

$\frac{39y}{39}=\frac{117}{39}$

$y=3$

2. Solve using elimination $\begin{cases} 5(2x+6y=-12) \\ -10x-5y=10 \end{cases}$

$10x+30y=-60$

$-10x-5y=10$

$0x+25y=-50$

$\frac{25y}{25}=\frac{-50}{25}$

$y=-2$

$2x+6(-2)=-12$

$2x-12=-12$

$+12 \quad +12$

$\frac{2x}{2}=\frac{0}{2}$

$x=0$

$\boxed{(0,-2)}$

When solving a system of word problems, follow a 4-step method.

4-Step Method:

1. Define variables
2. Write the system of equations
3. Solve showing all steps
4. State your solution in sentence form

1.) You are selling tickets for a high school basketball game. Student tickets cost \$3 and general admission tickets cost \$5. You sell 350 tickets and collect \$1450. How many of each type of ticket did you sell?

Define variables:
s = # of student tickets
g = # of gen. admin. tickets.

Systems of equations:
$s+g=350$
$3s+5g=1450$

Solve the system showing **all** steps:

$s+g=350$

$-s \quad -s$

$g=-s+350$

$3s+5(-s+350)=1450$

$3s-5s+1750=1450$

$-2s+1750=1450$

$-1750 \quad -1750$

$\frac{-2s}{-2}=\frac{-300}{-2}$

$s=150$

$150+g=350$

$-150 \quad -150$

$g=200$

State your solution:
We sold 150 student tickets and 200 general admission tickets.

2.) Missy Elliot loves to shop at Neiman Marcus. Last Saturday she bought pants and shirts for her new video. Each shirt cost \$125 and each pair of pants cost \$225. She came home with 26 items and spent exactly \$4950. How many pants and shirts did Missy buy?

Define variables:
s = # of shirts
p = # of pants

Systems of equations:
$s+p=26$
$125s+225p=4950$

Solve the system showing **all** steps:

$s+p=26$

$-s \quad -s$

$p=-s+26$

$125s+225(-s+26)=4950$

$125s-225s+5850=4950$

$-100s+5850=4950$

$-5850 \quad -5850$

$\frac{-100s}{-100}=\frac{-900}{-100}$

$s=9$

$9+p=26$

$-9 \quad -9$

$p=17$

State your solution:
Missy Elliot bought 9 shirts and 17 pairs of pants.

3.) You are in charge of decorating the gym for the Homecoming dance. You purchased 6 bags of balloons and 5 bags of large sparkling hanging stars all for $19.20. You soon realized that this was not enough to decorate the entire gym. On your second trip to the store, you bought 8 bags of balloons and 2 bags of large sparkling hanging stars all for $15.80. What was the price for each item?

Define variables:
b= price of balloons
s= price of stars

Systems of equations:
$6b+5s=19.20$
$8b+2s=15.80$

State your solution:
Each bag of balloons costs $1.45 and each star costs $2.10.

Solve the system showing **all** steps:

$-2(6b+5s=19.20)=-12b-10s=-38.40$

$5(8b+2s=15.80)=40b+10s=79.00$

$\frac{28b}{28}=\frac{40.60}{28}$

$b=1.45$

$8(1.45)+2s=15.80$

$11.60+2s=15.80$

$-11.60 \quad -11.60$

$\frac{2s}{2}=\frac{4.20}{2}$

$s=2.10$

4.) K-Mart had a sale on DVD's and video tapes for Labor Day weekend. Katie bought 2 tapes and 3 DVD's and spent $134. Emily bought 1 tape and 5 DVD's and spent $179. How much does each tape and DVD cost?

Define variables:

Systems of equations:

State your solution:

Solve the system showing **all** steps:

5.) You sell tickets for admission to your school play and collect a total of $104. Admission prices are $6 for adults and $4 for children. You sold 21 tickets. How many adult tickets and how many children tickets did you sell?

Define variables:

Systems of equations:

State your solution:

Solve the system showing **all** steps: