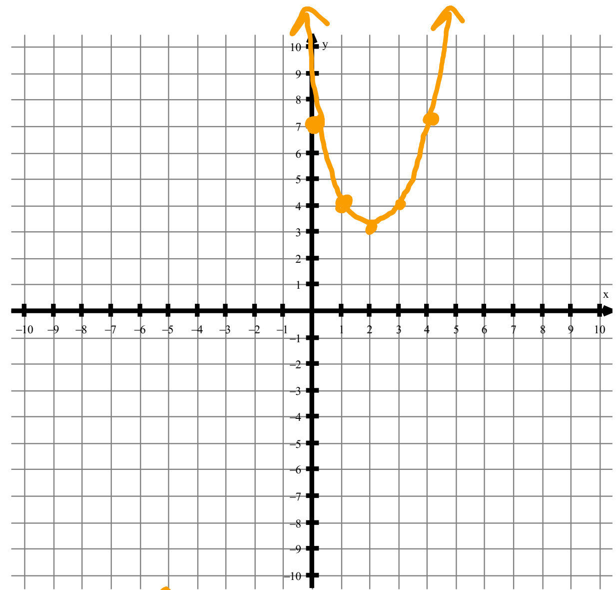
NOTES: ALGEBRA 2

4.1 FUNCTION NOTATION

STARTER:

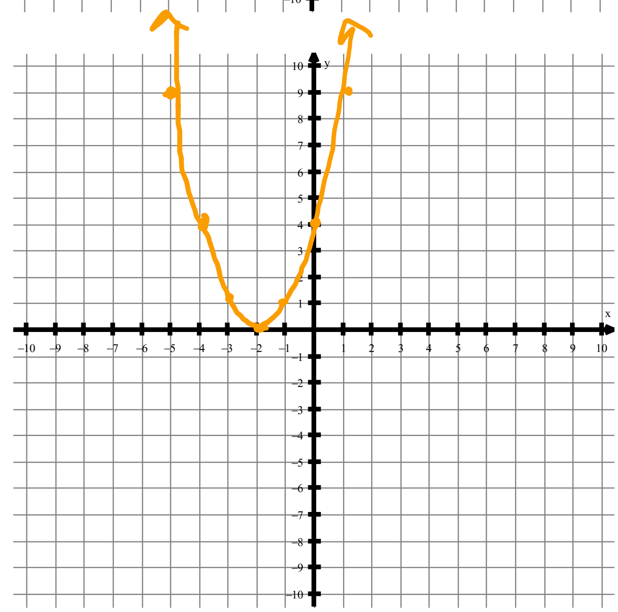
1. Graph $f(x)=(x-2)^2+3$ by completing the table.

x	$f(x)=(x-2)^2+3$	(x,y)
-1	$(-1-2)^2+3=(-3)^2+3=9+3=12$	(-1, 12)
0	$(0-2)^2+3=(-2)^2+3=4+3=7$	(0, 7)
1	$(1-2)^2+3=(-1)^2+3=1+3=4$	(1, 4)
2	$(2-2)^2+3=(0)^2+3=0+3=3$	(2, 3)
3	$(3-2)^2+3=(1)^2+3=1+3=4$	(3, 4)
4	$(4-2)^2+3=(2)^2+3=4+3=7$	(4, 7)
5	$(5-2)^2+3=(3)^2+3=9+3=12$	(5, 12)



2. Graph $f(x)=x^2+4x+4$

x	$f(x)=x^2+4x+4$	(x,y)
--5	$(-5)^2+4(-5)+4=25-20+4=9$	(-5, 9)
-4	$(-4)^2+4(-4)+4=16-16+4=4$	(-4, 4)
-3	$(-3)^2+4(-3)+4=9-12+4=1$	(-3, 1)
-2	$(-2)^2+4(-2)+4=4-8+4=0$	(-2, 0)
-1	$(-1)^2+4(-1)+4=1-4+4=1$	(-1, 1)
0	$(0)^2+4(0)+4=0+0+4=4$	(0, 4)
1	$(1)^2+4(1)+4=1+4+4=9$	(1, 9)



A Brief Review of Function Notation

We will be using **function notation** more often. The equation *y = 3x +7* can also be written as *f(x) = 3x + 7*

FUNCTION NOTATION $y=f(x)$

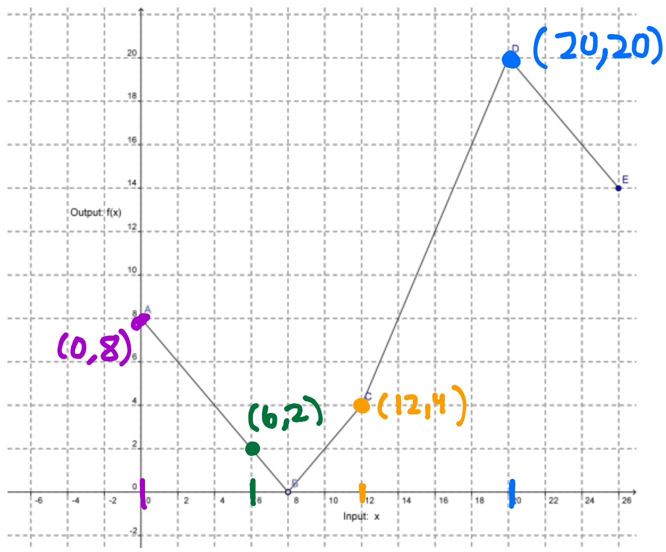
We say "f is a function of x" This means that y depends on x.

Sometimes, we refer to f(x) as "y"

In this case, the parenthesis do not mean multiplication. The x represents the number we are "plugging in" to the function. $f(x)$ represents what we will be "getting out."

- *f(2)* means Evaluate f(x) when x=2
- We don't have to name our functions *f*, any letter will do!

Example: This is a graph of a function. Think of it as a map.



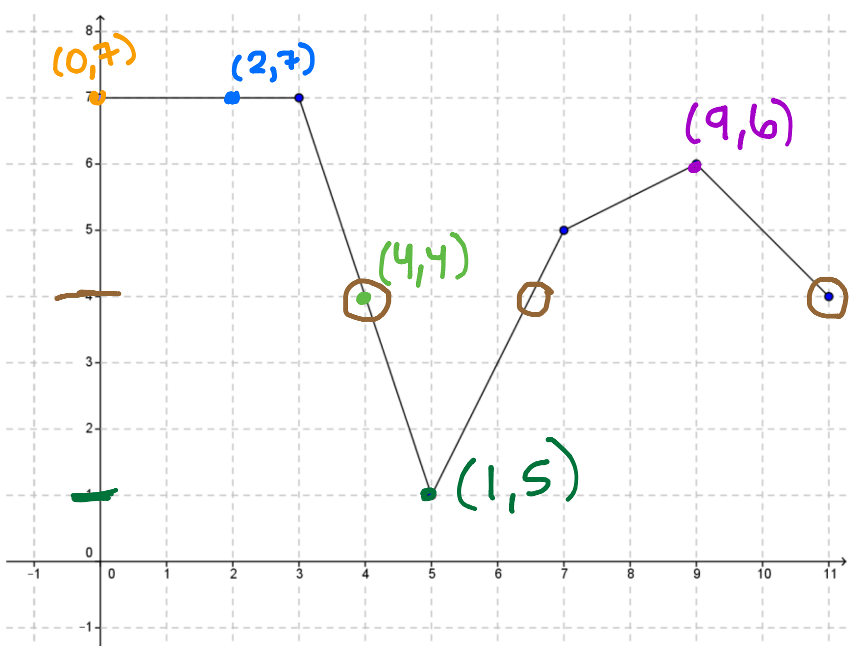
If you have to find $f(6)$, 6 is what we are plugging in. So, we look for 6 on the _x_-axis. We then find where our graph is at 6. $f(6) =$ _2_

$f(12)=$ _4_

$f(0) =$ _8_

$f(20) =$ _20_

Example: Use the graph below to answer the following questions:



a. $f(2) =$ 7

b. $f(0) =$ 7

c. $f(4) =$ 4

d. $f(9) =$ 6

e. If $f(x) = 1$, what is x? x = 5

f. If $f(x) = 4$, what is x? There are 3 answers…

x = 4, x = 6.5 x = 11

Example: Consider the function $g(x) = 2x - 5$.

1. Find $g(1)$, $g(-3)$, and $g(0)$.

$g(1) = 2(1) - 5$
$= 2 - 5$
$= -3$
$g(1) = -3$

$g(-3) = 2(-3) - 5$
$= -6 - 5$
$= -11$
$g(-3) = -11$

$g(0) = 2(0) - 5$
$= 0 - 5$
$= -5$
$g(0) = -5$

2. For what value of x does $g(x) = 7$?

$7 = 2x - 5$
$+5 \quad +5$
$\frac{12}{2} = \frac{2x}{2}$
$x = 6$

Example: Given $f(x) = 2x + 3$.

1. Find the values of $f(0)$, $f(7)$, $f(-5)$.

$f(0) = 2(0) + 3$
$= 0 + 3$
$= 3$
$f(0) = 3$

$f(7) = 2(7) + 3$
$= 14 + 3$
$= 17$
$f(7) = 17$

$f(-5) = 2(-5) + 3$
$= -10 + 3$
$= -7$
$f(-5) = -7$

2. For what value of x does $f(x) = 10$?

$10 = 2x + 3$
$-3 \quad -3$
$\frac{7}{2} = \frac{2x}{2}$
$x = \frac{7}{2}$

Example: Evaluate the following given $f(x) = 3x^2 - 4x + 7$ and $g(x) = 2(x-3)^2 + 5$

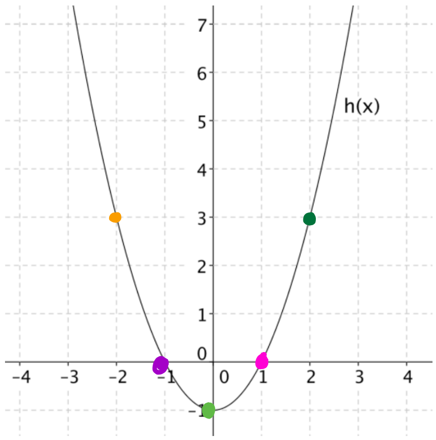
1. $f(-1) = 3(-1)^2 - 4(-1) + 7$
$= 3(1) - 4(-1) + 7$
$= 3 + 4 + 7$
$= 14$
$f(-1) = 14$

2. $f(0) = 3(0)^2 - 4(0) + 7$
$= 3(0) - 4(0) + 7$
$= 0 + 0 + 7$
$= 7$
$f(0) = 7$

3. $g(4) = 2(4-3)^2 + 5$
$= 2(1)^2 + 5$
$= 2(1) + 5$
$= 2 + 5$
$= 7$
$g(4) = 7$

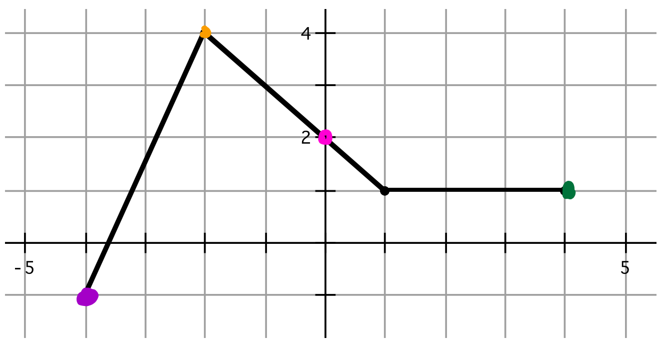
4. $g(1) = 2(1-3)^2 + 5$
$= 2(-2)^2 + 5$
$= 2(4) + 5$
$= 8 + 5$
$= 13$
$g(1) = 13$

Example: The graph of the function $h(x)$ is drawn below. Use it to find the following:



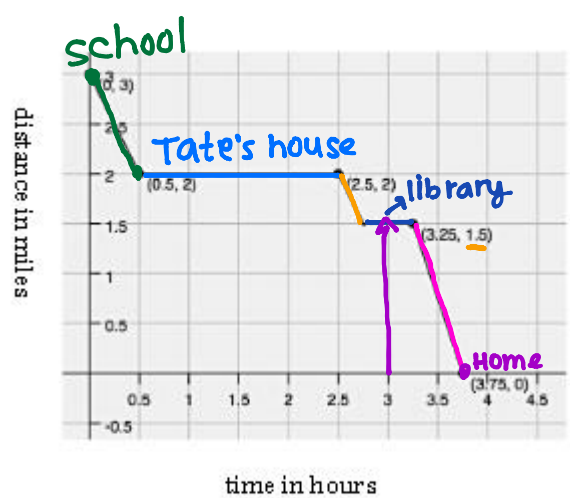
1. $h(2) =$ 3
2. $h(-2) =$ 3
3. $h(-1) =$ 0
4. If $h(x) = -1$, $x =$ 0
5. If $h(x) = 0$, $x =$ -1 OR $x =$ 1

Example: Use the graph of f(x) below to evaluate the following.



1. $f(4) =$ 1
2. $f(-2) =$ 4
3. $f(0) =$ 2
4. $f(x) = -1$, $x =$ -4

Example: Isaac lives 3 miles away from his school. School ended at 3 pm and Isaac began his walk home with his friend Tate who lives 1 mile from the school in the direction of Isaac's house. Isaac stayed at Tate's house for a while and then started home. On the way, he stopped at the library then he hurried home. The graph at the below is the **piece-wise defined function** that shows Isaac's distance from home during the time it took him to arrive home.



1. How much time passed between school ending and Isaac's arrival home? It took Isaac 3hours and 45 minutes to get home.

2. How long did Isaac stay at Tate's house?
Isaac stayed at Tate's house for 2 hours.

3. How far is the library from Isaac's house?
The library is 1.5 miles from Isaac's house.

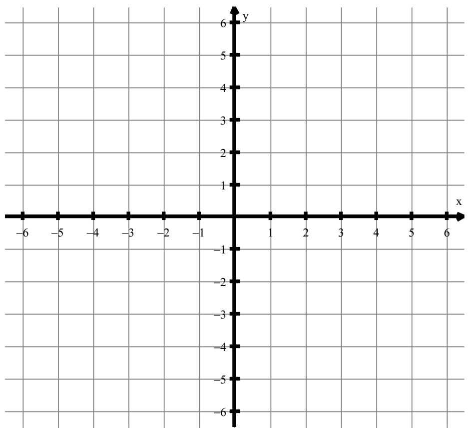
4. Where was Isaac 3 hours after school ended?
Isaac was at the library 3 hours after school ended.

5. Rewrite #4 in function notation.
What is f(3)?

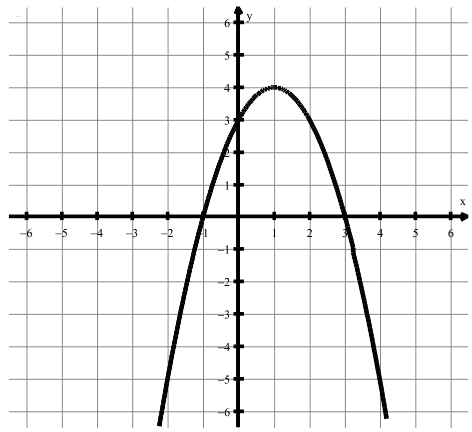
NOTES: ALGEBRA 2
4.2 PROPERTIES OF QUADRATICS

STARTER:

1. Graph the line $f(x) = \frac{-2}{3}x + 4$



2. Find the domain and range of the following function.



3. Given $f(x) = (x+1)^2 - 5$, find $f(-4)$

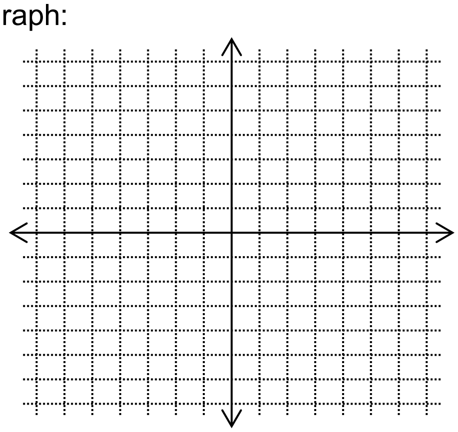
4. Solve for x. $6y - \frac{2}{3}x = 4$

<u>INTRODUCTION TO QUADRATIC FUNCTIONS</u>

Recall: The Quadratic Parent Function

Equation: ______________

Graph:



Description of Graph:

General Equation:

Vertex Form:

Intercept Form:

Domain:

Range:

DETERMINE WHETHER THE GIVEN FUNCTION IS LINEAR, QUADRATIC, OR NEITHER.

1. $f(x) = 17x - 9$

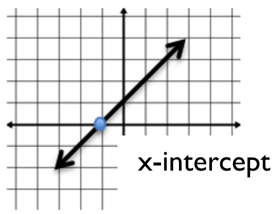
2. $g(x) = 2x^2 - 3x + 1$

3. $f(x) = x^3 + 3x - 2$

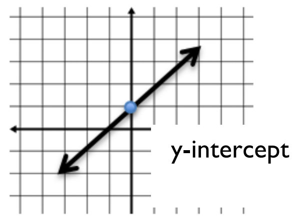
KEY FEATURES OF A QUADRATIC

- The **x-intercept (also called a zero)** is the point where a graph crosses or touches the *x*-axis.
  - It is the ordered pair _______, where *x* is a real number.
  - The **x-intercept** is where ________.

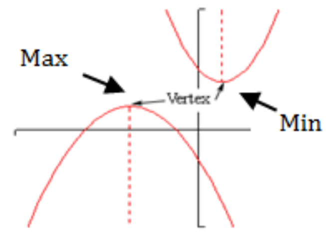
- The **y-intercept** is the point where a graph crosses or touches they *y*-axis.
  - It is the ordered pair _______, where *y* is a real number.
  - The **y-intercept** is where _________.



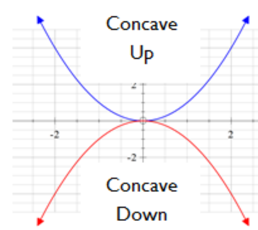
____



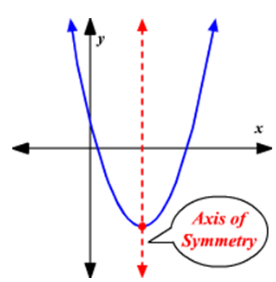
- The **vertex** of a parabola is the high point or low point in the curve. The **vertex** also represents the **minimum** or **maximum** value of the parabola.
-
  - A **maximum** occurs when
  - A **minimum** occurs when



- The parabola will be:
  - **concave up** if
  - **concave down** if

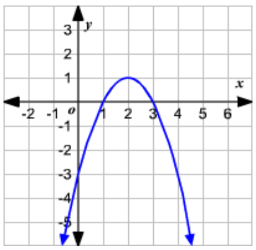
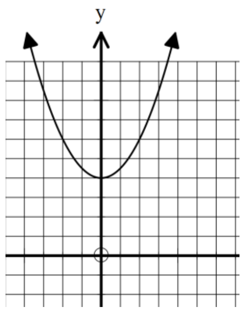
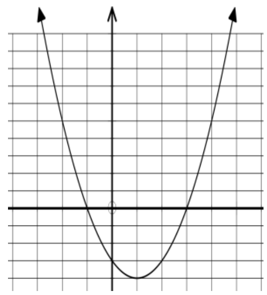
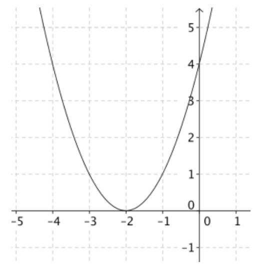


- Every parabola has an **axis of symmetry** which is the line that runs down its 'center.'
  - The **axis of symmetry** runs through the ___________
  - The equation for the **axis of symmetry** is:



Finding Key Points from a Graph:

Example: For each graph, find the **x-intercept(s)** and **y-intercept** and the **vertex**. Is the vertex a **maximum** or a **minimum**? Draw a dotted line for the **axis of symmetry** and write the equation for the axis of symmetry.

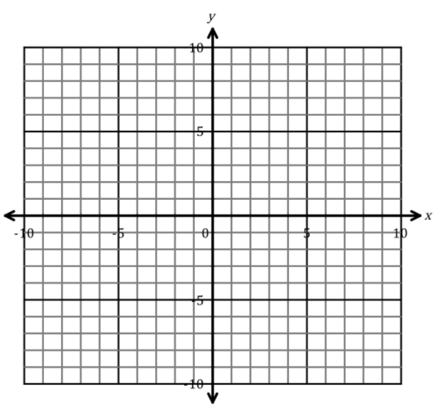
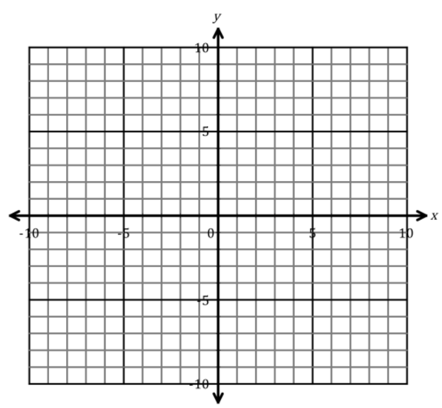
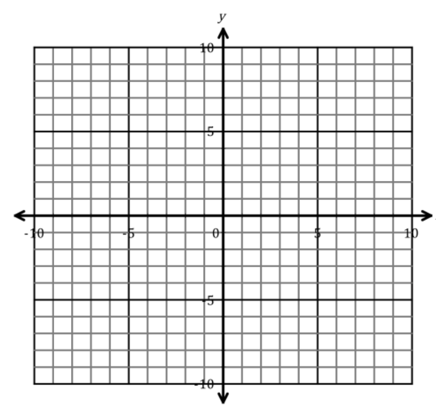
y-intercept:	y-intercept:	y-intercept:	y-intercept:
x-intercept(s):	x-intercept(s):	x-intercept(s):	x-intercept(s):
vertex:	vertex:	vertex:	vertex:
MAX or MIN	MAX or MIN	MAX or MIN	MAX or MIN
axis of symmetry:	axis of symmetry:	axis of symmetry:	axis of symmetry:

Example: Sketch the graph of the quadratic functions given the key features. Label the key features on your graph. Draw a dotted line for the axis of symmetry.

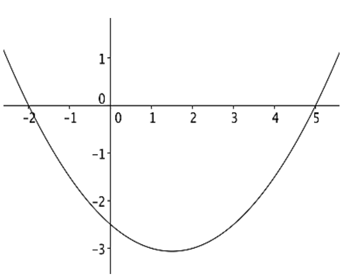
1. *x*-intercepts: (1, 0) & (5, 0)
   y-intercept: (0, 10)
   minimum: (3, -8)

2. *x*-intercept: (-3, 0)
   y-intercept: (0, -6)
   maximum: (-3, 0)

3. *x*-intercept: none
   y-intercept: (0, 4)
   minimum: (2, 2)

You are presented with the graph below. How could you find the *exact* line of the **axis of symmetry**? Will this method always work?



Finding Key Points from a Table

Find the **minimum point (vertex)** of the function given a table of values.

1. Look for the lowest *y*-value.
2. Check if the lowest *y*-value is "evenly sandwiched" between the same value.

Find the **maximum point (vertex)** of the function given a table of values.

1. Look for the highest *y*-value.
2. Check if the highest *y*-value is "evenly sandwiched" between the same value.

For the following examples, use the table to identify the x and y intercepts and the vertex. Is the vertex a maximum or a minimum? Write an equation for the axis of symmetry.

1.

x	*y*
0	5
1	0
2	-3
3	-4
4	-3
5	0
6	5

y-intercept:

x-intercept(s):

vertex:

MAX or MIN

concave up or down?

axis of symmetry:

2.

x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

y-intercept:

x-intercept(s):

vertex:

MAX or MIN

concave up or down?

axis of symmetry:

3.

x	*y*
-2	1
-1	6
0	9
1	10
2	9
3	6
4	1

y-intercept:

x-intercept(s):

vertex:

MAX or MIN

concave up or down?

axis of symmetry:

Finding Key Points from the Equation

When given an equation, we can find the key points algebraically. We will discuss x-intercepts next unit but for now:

- To find the **y-intercept**, we
- To find the **x-intercept(s)**, we

<u>Practice:</u> Given the equation, find the y-intercept.

1. $y = x^2 \quad 9$

y-intercept:

2. $y = x^2$

y-intercept:

3. $y = 3(x \quad 1)^2 + 7$

y-intercept:

4. $y = \quad 16x^2 + 59x + 3$

y-intercept:

NOTES: ALGEBRA 2
4.3 GRAPHING QUADRATICS BY HAND

STARTER:

For the following examples, identify the key features of the quadratic.

1. $f(x) = 5x - 3$

x-intercept:

y-intercept:

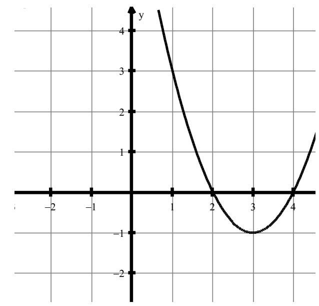
2.

x	y
-3	-16
-2	-6
-1	0
0	2
1	0
2	-6

x-intercept:
y-intercept:
vertex:
MAX or MIN:

Concave up or down?
Axis of Symmetry:

3.



x-intercept:
y-intercept:
vertex:
MAX or MIN:

Concave up or down?
Axis of Symmetry:

4. Given $f(x) = 2x^2 + 3x - 4$, find $f(-1)$.

5. $\left(\frac{-2}{3}\right)^2 =$

GRAPHING QUADRATIC FUNCTIONS

We have talked about identifying key features from a graph, table, and equation. Today we will discuss graphing quadratic functions by hand from the equation. First, let's review standard form of a quadratic function:

STANDARD FORM OF A QUADRATIC FUNCTION:

$$f(x) = ax^2 + bx + c$$

where *a, b,* and *c* are integers and a≠0.

Example: For the following functions, identify A, B, C. Is the quadratic function in standard form?

1. $f(x) = 2x^2 - 6x + 5$

A=____ B=____ C=____

Standard form?

2. $g(x) = x^2 - \frac{2}{3}x - 3$

A=____ B=____ C=____

Standard form?

3. $h(x) = -3x^2 - 4$

A=____ B=____ C=____

Standard form?

All parabolas have an **axis of symmetry**. If you were to "fold" a parabola along its axis of symmetry, the points on either side of this line would match.

AXIS OF SYMMETRY:

The axis of symmetry is a vertical line that goes through the middle of the parabola and splits the parabola into equal parts. The parabola "mirrors" itself on either side of the axis of symmetry.

$$x = \frac{-b}{2a}$$

Example: Find the axis of symmetry in the following quadratic functions.

a. $f(x) = -x^2 + 2x + 8$ b. $g(x) = x^2 - 8x + 15$ c. $y = x^2 + 2x - 3$

d. $h(x) = 2x^2 - 4x + 1$ e. $y = -x^2 - 2x + 5$ c. $h(x) = x^2 - 3x + 1$

The maximum or minimum point on a parabola is called the **vertex**.

VERTEX:

Plug it in,
Plug it in!

The vertex is where the axis of symmetry and the parabola intersect. The x-coordinate is the axis of symmetry $\frac{-b}{2a}$. To find the y-coordinate, plug in $x = \frac{-b}{2a}$ and solve for y.

Example: Find the vertex in the following quadratic functions.

a. $f(x) = -x^2 + 2x + 8$ b. $g(x) = x^2 - 8x + 15$ c. $y = x^2 + 2x - 3$

d. $h(x) = 2x^2 - 4x + 1$ e. $y = -x^2 - 2x + 5$ f. $h(x) = x^2 - 3x + 1$

STEPS TO GRAPHING A QUADRATIC IN STANDARD FORM:

1. Find the **axis of symmetry** by using $x = \frac{b}{2a}$.
2. Identify the **vertex**. Remember, the x coordinate of the vertex is the same as the axis of symmetry
3. Make a table and find 2 points on either side of the vertex.
4. Plot points.
5. Connect points, making sure your graph forms a parabola

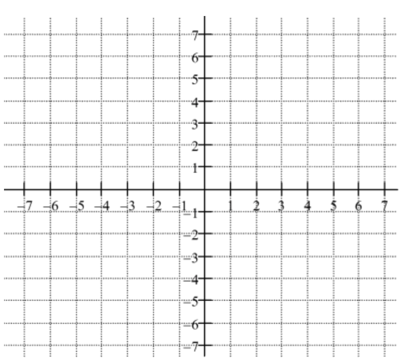
Example: Complete the table and graph the function. Identify the axis of symmetry and the vertex. State the domain and range.

a. $f(x)=x^2+4x \quad 5$

x	$x^2+4x \quad 5$	(x,y)

Axis of symmetry:

Vertex: (,)



Domain = ____________

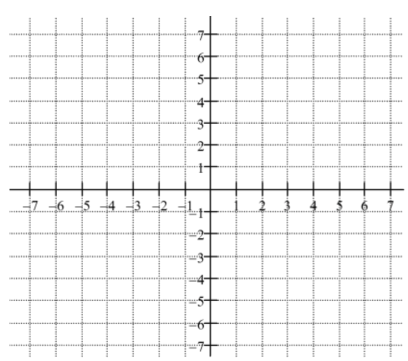
Range = _____________

b. $g(x)=x^2-8x+15$

x	$x^2-8x+15$	(x,y)

Axis of symmetry:

Vertex: (,)



Domain = ____________

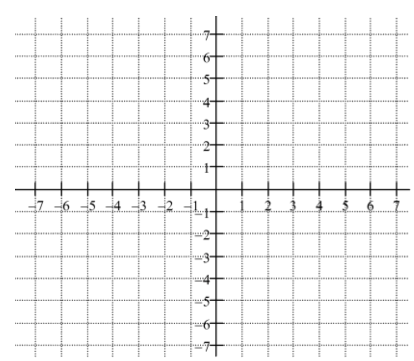
Range = _____________

c. $y=x^2+2x-3$

x	x^2+2x-3	(x,y)

Axis of symmetry:

Vertex: (,)

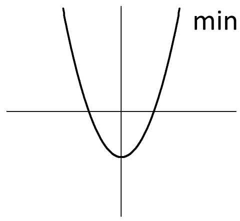


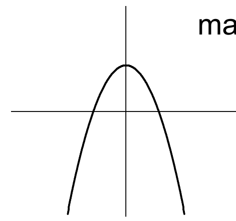
Domain = ____________

Range = _____________

Maximum and minimum values:

The y-coordinate of the vertex is the maximum value or the minimum value. When the graph opens up, there is a minimum value. When the graph opens down, there is a maximum value.





Example: Determine whether the function has a maximum or minimum value. State the maximum or minimum value of each.

a. $f(x) = -x^2 + 2x + 8$

b. $g(x) = x^2 - 8x + 15$

c. $y = x^2 + 2x - 3$

Example: Fred jumps feet first from a diving board, springing up into the air and then dropping feet-first. The distance *d* in feet from his feet to the pool's surface t seconds after he jumps is $d(t) = -16t^2 + 18t + 2$.

a) Draw a picture of what Fred's jump would look like.

b) When does the maximum height occur?

c) What is the maximum height of Fred's feet during this jump?

d) What does the constant term ***2*** in the equation tell you about Fred's jump?