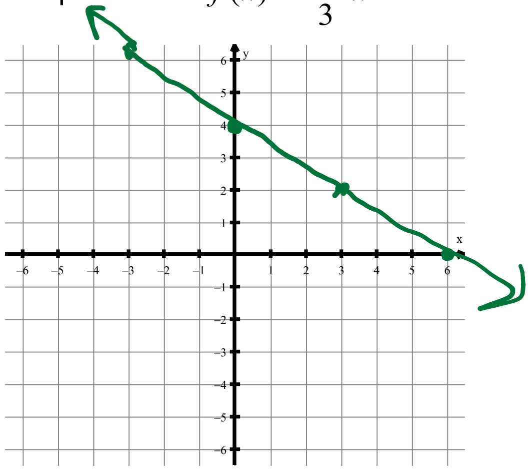
NOTES: ALGEBRA 2

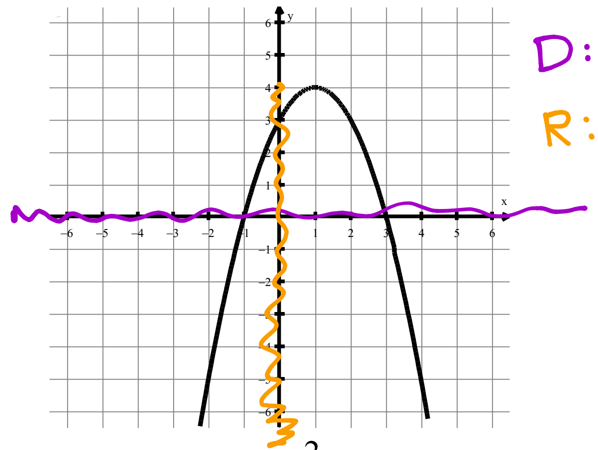
4.2 PROPERTIES OF QUADRATICS

STARTER:

1. Graph the line $f(x) = \frac{-2}{3}x + 4$



2. Find the domain and range of the following function.



D: $(-\infty, \infty)$

R: $(-\infty, 4]$

3. Given $f(x) = (x+1)^2 - 5$, find $f(-4)$

$f(-4) = (-4+1)^2 - 5$

$= (-3)^2 - 5$

$= 9 - 5$

$= 4$

$\boxed{f(-4) = 4}$

4. Solve for x. $6y - \frac{2}{3}x = 4$

$-6y \qquad -6y$

$3\left(\frac{-2}{3}x = -6y + 4\right)$

$\frac{-2x}{-2} = \frac{-18y + 12}{-2}$

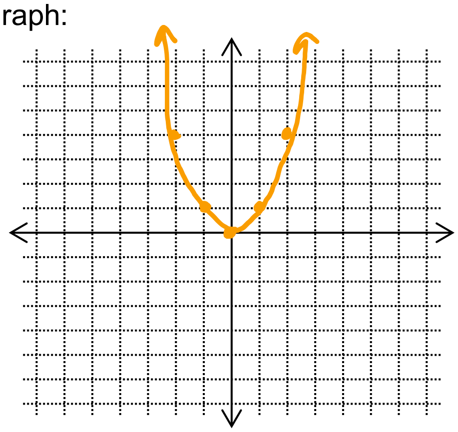
$\boxed{x = 9y - 6}$

INTRODUCTION TO QUADRATIC FUNCTIONS

Recall: The Quadratic Parent Function

Equation: $y = x^2$

Graph:



Description of Graph: "U" shape "∩" shape

General Equation: $y = ax^2 + bx + c$

Vertex Form: $y = a(x-h)^2 + k$

Intercept Form: $y = a(x-p)(x-q)$

Domain: D: $(-\infty, \infty)$

Range: R: $[0, \infty)$

DETERMINE WHETHER THE GIVEN FUNCTION IS LINEAR, QUADRATIC, OR NEITHER.

1. $f(x) = 17x - 9$

Linear

2. $g(x) = 2x^2 - 3x + 1$

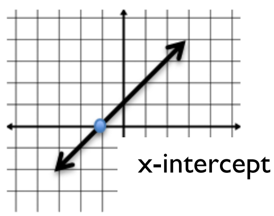
Quadratic

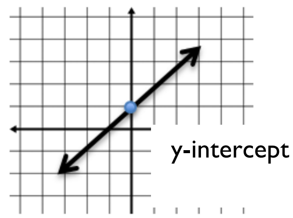
3. $f(x) = x^3 + 3x - 2$

Neither

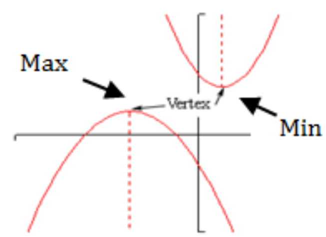
KEY FEATURES OF A QUADRATIC

- The **x-intercept (also called a zero)** is the point where a graph crosses or touches the *x*-axis.
    - It is the ordered pair $(x, 0)$, where *x* is a real number.
    - The **x-intercept** is where $y = 0$.
- The **y-intercept** is the point where a graph crosses or touches they *y*-axis.
    - It is the ordered pair $(0, y)$, where *y* is a real number.
    - The **y-intercept** is where $x = 0$.



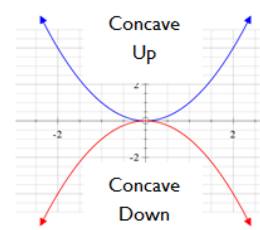


- The **vertex** of a parabola is the high point or low point in the curve. The **vertex** also represents the **minimum** or **maximum** value of the parabola.
-
    - A **maximum** occurs when There is a peak "∩" ← max
    - A **minimum** occurs when There is a valley "U" ← min



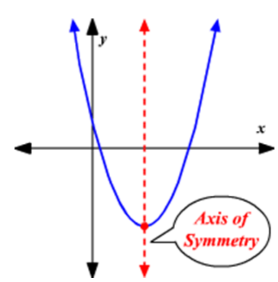
$y = \underline{a}x^2 + bx + c$

- The parabola will be:
    - **concave up** if if "a" is positive. "U"
    - **concave down** if if "a" is negative "∩"



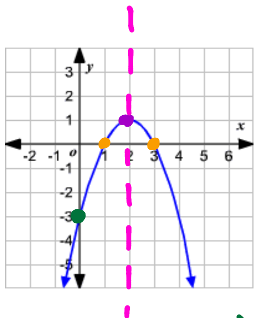
- Every parabola has an **axis of symmetry** which is the line that runs down its 'center.'
    - The **axis of symmetry** runs through the vertex
    - The equation for the **axis of symmetry** is:

$x =$



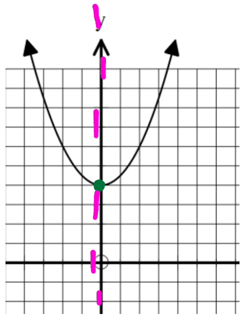
Finding Key Points from a Graph:

Example: For each graph, find the **x-intercept(s)** and **y-intercept** and the **vertex**. Is the vertex a **maximum** or a **minimum**? Draw a dotted line for the **axis of symmetry** and write the equation for the axis of symmetry.



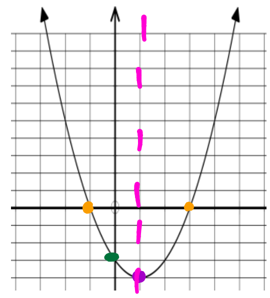
y-intercept: (0,-3)
x-intercept(s): (1,0) & (3,0)
vertex: (2,1)
MAX or MIN

axis of symmetry:
x=2



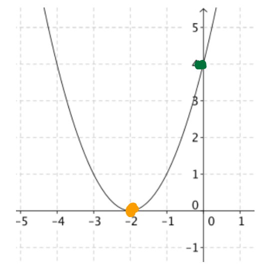
y-intercept: (0,4)
x-intercept(s): none
vertex: (0,4)
MAX or MIN

axis of symmetry:
x=0



y-intercept: (0,-3)
x-intercept(s): (-1,0) & (3,0)
vertex: (1,-4)
MAX or MIN

axis of symmetry:
x=1

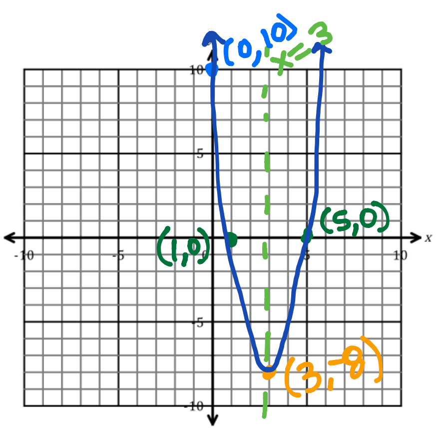


y-intercept: (0,4)
x-intercept(s): (-2,0)
vertex: (-2,0)
MAX or MIN

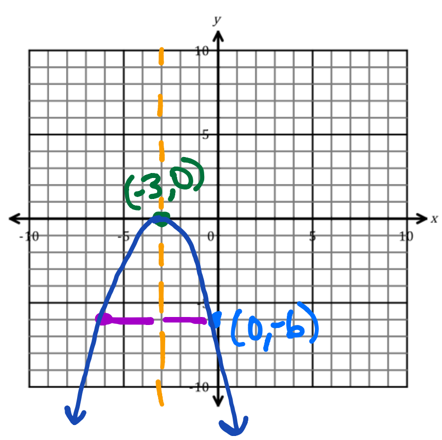
axis of symmetry:
x=-2

Example: Sketch the graph of the quadratic functions given the key features. Label the key features on your graph. Draw a dotted line for the axis of symmetry.

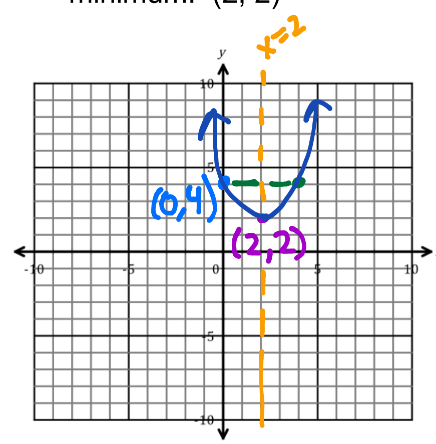
1. *x*-intercepts: (1, 0) & (5, 0)
   y-intercept: (0, 10)
   minimum: (3, -8)



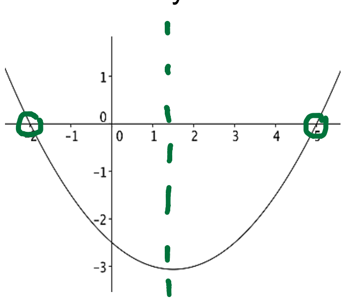
2. *x*-intercept: (-3, 0)
   y-intercept: (0, -6)
   maximum: (-3, 0)



3. *x*-intercept: none
   y-intercept: (0, 4)
   minimum: (2, 2)



You are presented with the graph below. How could you find the *exact* line of the **axis of symmetry**? Will this method always work?



$$\frac{5+(-2)}{2} = \frac{3}{2}$$

$$\boxed{x = \frac{3}{2}}$$

or $\boxed{x = 1.5}$

Finding Key Points from a Table

Find the **minimum point (vertex)** of the function given a table of values.

1. Look for the lowest *y*-value.
2. Check if the lowest *y*-value is "evenly sandwiched" between the same value.

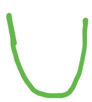
Find the **maximum point (vertex)** of the function given a table of values.

1. Look for the highest *y*-value.
2. Check if the highest *y*-value is "evenly sandwiched" between the same value.

For the following examples, use the table to identify the x and y intercepts and the vertex. Is the vertex a maximum or a minimum? Write an equation for the axis of symmetry.

1.

x	*y*
0	5
1	0
2	-3
3	-4
4	-3
5	0
6	5



y-intercept: (0,5)

x-intercept(s): (1,0) & (5,0)

vertex: (3,-4)

MAX or MIN

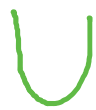
concave up or down?

axis of symmetry:

x=3

2.

x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



y-intercept: (0,0)

x-intercept(s): (0,0)

vertex: (0,0)

MAX or MIN

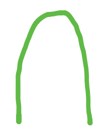
concave up or down?

axis of symmetry:

x=0

3.

x	*y*
-2	1
-1	6
0	9
1	10
2	9
3	6
4	1



y-intercept: (0,9)

x-intercept(s): none

vertex: (1,10)

MAX or MIN

concave up or down?

axis of symmetry:

x=1

Finding Key Points from the Equation

When given an equation, we can find the key points algebraically. We will discuss x-intercepts next unit but for now:

- To find the **y-intercept**, we plug in x=0
- To find the **x-intercept(s)**, we plug in y=0

Practice: Given the equation, find the y-intercept.

1. $y = x^2 - 9$

$y = (0)^2 - 9$
$= 0 - 9$
$= -9$

y-intercept: (0,-9)

2. $y = x^2$

$y = 0^2$
$= 0$

y-intercept: (0,0)

3. $y = 3(x-1)^2 + 7$

$y = 3(0-1)^2 + 7$
$= 3(-1)^2 + 7$
$= 3 + 7 = 10$

y-intercept: (0,10)

4. $y = -16x^2 + 59x + 3$

$y = -16(0)^2 + 59(0) + 3$
$= 0 + 0 + 3$
$= 3$

y-intercept: (0,3)