NOTES: ALGEBRA 2
4.3 GRAPHING QUADRATICS BY HAND

STARTER:

For the following examples, identify the key features of the quadratic.

1. $f(x) = 5x - 3$

x-intercept: $(\frac{3}{5}, 0)$

$0 = 5x - 3$
$+3 \quad\quad +3$
$\frac{3}{5} = \frac{5x}{5} \quad x = \frac{3}{5}$

y-intercept: $(0, -3)$

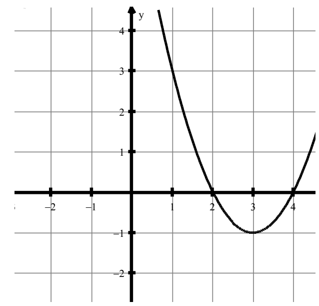
$f(0) = 5(0) - 3$
$= 0 - 3$
$= -3$

2.

x	y
-3	-16
-2	-6
-1	0
0	2
1	0
2	-6

x-intercept: $(-1, 0)$ & $(1, 0)$
y-intercept: $(0, 2)$
vertex: $(0, 2)$
MAX or MIN:
max value: 2
Concave up or down?
Axis of Symmetry: $x = 0$

3.



x-intercept: $(2, 0)$ & $(4, 0)$
y-intercept: N/A
vertex: $(3, -1)$
MAX or MIN
min value: -1
Concave up or down?
Axis of Symmetry: $x = 3$

4. Given $f(x) = 2x^2 + 3x - 4$, find $f(-1)$.

$f(-1) = 2(-1)^2 + 3(-1) - 4$
$= 2(1) + 3(-1) - 4$
$= 2 - 3 - 4$
$= -5$

$\boxed{f(-1) = -5}$

5. $\left(\frac{-2}{3}\right)^2 =$

$\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \boxed{\frac{4}{9}}$

GRAPHING QUADRATIC FUNCTIONS

We have talked about identifying key features from a graph, table, and equation. Today we will discuss graphing quadratic functions by hand from the equation. First, let's review standard form of a quadratic function:

STANDARD FORM OF A QUADRATIC FUNCTION:

$$f(x) = ax^2 + bx + c$$

where a, b, and c are integers and a≠0.

Example: For the following functions, identify A, B, C. Is the quadratic function in standard form?

1. $f(x) = 2x^2 - 6x + 5$

A= 2 B= -6 C= 5

Standard form? yes

2. $g(x) = x^2 - \frac{2}{3}x - 3$

A= 1 B= $-\frac{2}{3}$ C= -3

Standard form? NO

3. $h(x) = -3x^2 - 4$

$-3x^2 + 0x - 4$

A= -3 B= 0 C= -4

Standard form? yes

All parabolas have an **axis of symmetry**. If you were to "fold" a parabola along its axis of symmetry, the points on either side of this line would match.

AXIS OF SYMMETRY:

The axis of symmetry is a vertical line that goes through the middle of the parabola and splits the parabola into equal parts. The parabola "mirrors" itself on either side of the axis of symmetry.

$$\boxed{x = \frac{-b}{2a}}$$

Example: Find the axis of symmetry in the following quadratic functions.

a. $f(x) = -x^2 + 2x + 8$

$a=-1 \quad b=2 \quad c=8$

$x = \frac{-b}{2a} = \frac{-2}{2(-1)} = \frac{-2}{-2} = 1$ $\boxed{x=1}$

b. $g(x) = x^2 - 8x + 15$

$x = \frac{-b}{2a} = \frac{8}{2(1)} = \frac{8}{2} = 4$ $\boxed{x=4}$

c. $y = x^2 + 2x - 3$

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$

$\boxed{x=-1}$

d. $h(x) = 2x^2 - 4x + 1$

$x = \frac{-b}{2a} = \frac{4}{2(2)} = \frac{4}{4} = 1$

$\boxed{x=1}$

e. $y = -x^2 - 2x + 5$

$x = \frac{-b}{2a} = \frac{2}{2(-1)} = \frac{2}{-2} = -1$

$\boxed{x=-1}$

c. $h(x) = x^2 - 3x + 1$

$x = \frac{-b}{2a} = \frac{3}{2(1)} = \frac{3}{2}$

$\boxed{x=\frac{3}{2}}$

The maximum or minimum point on a parabola is called the **vertex**.

VERTEX:

Plug it in,
Plug it in!

The vertex is where the axis of symmetry and the parabola intersect. The x-coordinate is the axis of symmetry $\frac{-b}{2a}$. To find the y-coordinate, plug in $x = \frac{-b}{2a}$ and solve for y.

Example: Find the vertex in the following quadratic functions.

a. $f(x) = -x^2 + 2x + 8$

$x = 1$

$f(1) = -(1)^2 + 2(1) + 8$

$= -1 + 2 + 8$

$= 9$

$\boxed{(1,9)}$

b. $g(x) = x^2 - 8x + 15$

$x = 4$

$g(4) = (4)^2 - 8(4) + 15$

$= 16 - 32 + 15$

$= -16 + 15$

$= -1$ $\boxed{(4,-1)}$

c. $y = x^2 + 2x - 3$

$x = -1$

$y = (-1)^2 + 2(-1) - 3$

$= 1 - 2 - 3$

$= -4$

$\boxed{(-1,-4)}$

d. $h(x) = 2x^2 - 4x + 1$

$x = 1$

$h(1) = 2(1)^2 - 4(1) + 1$

$= 2 - 4 + 1$

$= -1$

$\boxed{(1,-1)}$

e. $y = -x^2 - 2x + 5$

$x = -1$

$y = -(-1)^2 - 2(-1) + 5$

$y = -1 + 2 + 5$

$y = 6$

$\boxed{(-1,6)}$

f. $h(x) = x^2 - 3x + 1$

$x = \frac{3}{2}$

$h(\frac{3}{2}) = (\frac{3}{2})^2 - 3(\frac{3}{2}) + 1$

$= \frac{9}{4} - \frac{9}{2} + 1$

$= \frac{9}{4} - \frac{18}{4} + \frac{4}{4} = \frac{-5}{4}$

$\boxed{(\frac{3}{2}, -\frac{5}{4})}$

STEPS TO GRAPHING A QUADRATIC IN STANDARD FORM:

1. Find the **<u>axis of symmetry</u>** by using $x = \frac{b}{2a}$.
2. Identify the **<u>vertex</u>**. Remember, the x coordinate of the vertex is the same as the axis of symmetry
3. Make a table and find <u>2 points on either side</u> of the vertex.
4. Plot points.
5. Connect points, <u>making sure your graph forms a parabola</u>

<u>Example:</u> Complete the table and graph the function. Identify the axis of symmetry and the vertex. State the domain and range.

a. $f(x) = x^2 + 4x - 5$

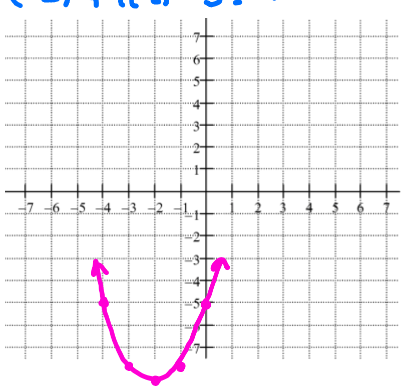
x	$x^2 + 4x - 5$	(x, y)
0	$0^2+4(0)-5$ $0-5=-5$	$(0,-5)$
-1	$(-1)^2+4(-1)-5$ $1-4-5=-8$	$(-1,-8)$
-2	-9	$(-2,-9)$
-3	$(-3)^2+4(-3)-5$ $9-12-5=-8$	$(-3,-8)$
-4	$(-4)^2+4(-4)-5$ $16-16-5=-5$	$(-4,-5)$

$x = \frac{-b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$

Axis of symmetry: $x = -2$

Vertex: (-2 , -9)

$y = (-2)^2 + 4(-2) - 5 = 4 - 8 - 5 = -9$



Domain = $(-\infty, \infty)$

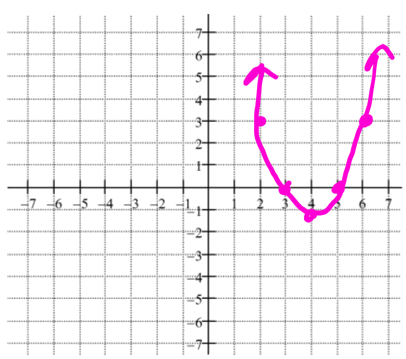
Range = $[-9, \infty)$

b. $g(x) = x^2 - 8x + 15$

x	$x^2 - 8x + 15$	(x, y)
2	$(2)^2-8(2)+15$ $4-16+15=3$	$(2,3)$
3	$(3)^2-8(3)+15$ $9-24+15=0$	$(3,0)$
4	-1	$(4,-1)$
5	$(5)^2-8(5)+15$ $25-40+15=0$	$(5,0)$
6	$(6)^2-8(6)+15$ $36-48+15=3$	$(6,3)$

Axis of symmetry: $x = 4$

Vertex: (4 , -1)



Domain = $(-\infty, \infty)$

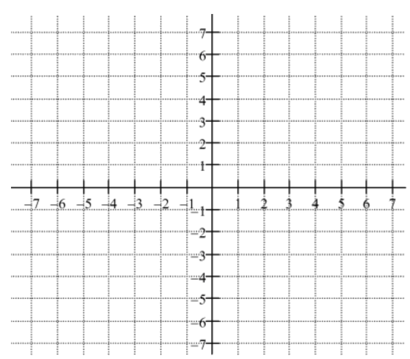
Range = $[-1, \infty)$

c. $y = x^2 + 2x - 3$

x	$x^2 + 2x - 3$	(x, y)

Axis of symmetry:

Vertex: (,)

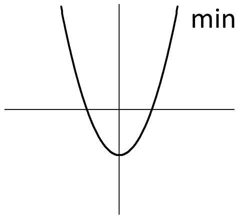


Domain = ____________

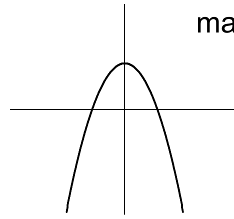
Range = ____________

Maximum and minimum values:

The y-coordinate of the vertex is the maximum value or the minimum value. When the graph opens up, there is a minimum value. When the graph opens down, there is a maximum value.



minimum



maximum

Example: Determine whether the function has a maximum or minimum value. State the maximum or minimum value of each.

a. $f(x) = -x^2 + 2x + 8$

Maximum

vertex: (1,9)

Max value: 9

b. $g(x) = x^2 - 8x + 15$

Minimum

vertex: (4,-1)

Min value: -1

c. $y = x^2 + 2x - 3$

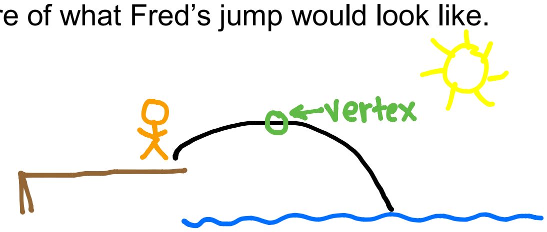
Minimum

vertex: (-1,-4)

Min value: -4

Example: Fred jumps feet first from a diving board, springing up into the air and then dropping feet-first. The distance *d* in feet from his feet to the pool's surface t seconds after he jumps is $d(t) = -16t^2 + 18t + 2$.

a) Draw a picture of what Fred's jump would look like.



b) When does the maximum height occur?

Axis of symmetry

$t = \frac{-b}{2a}$

$t = \frac{-18}{2(-16)} = \frac{-18}{-32} \approx 0.5625$

The maximum height occurs at 0.5625 seconds

c) What is the maximum height of Fred's feet during this jump?

Plug it in... Plug it in!

$-16(.5625)^2 + 18(.5625) + 2 = 7.9$

The maximum height of Fred's feet during the jump is 7.9 feet.

d) What does the constant term ***2*** in the equation tell you about Fred's jump?

The diving board is 2 feet off the ground.