Name: ____________________

Precalculus: UNIT 9
Lesson 1: Basic Counting and Permutations

The **Multiplication Principal of Counting** states that for independent events A and B, if event A can occur in m different ways and event B can occur in n different ways, then event A followed by event B can happen in $m \cdot n$ different ways.

Example 1: Suppose I have 6 shirts and 5 ties. How many shirt-and-tie outfits can I make?

$\underset{\text{shirt}}{\underline{6}}$ $\underset{\text{tie}}{\underline{5}}$ $= \boxed{30 \text{ outfits}}$

Example 2: In how many ways can we form a license plate if there are 7 characters, none of which is the letter O, the first of which a number (0-9), the second of which is a letter (not O), and the remaining 5 of which can either be a number or a letter (not O)?

$\underline{10}$ $\underline{25}$ $\underline{35}$ $\underline{35}$ $\underline{35}$ $\underline{35}$ $\underline{35}$

$\boxed{10 \cdot 25 \cdot 35^5 = 1.31E10}$

⇒ A **permutation** is the number of ways that a set of n objects (called an n-set) can be arranged **in order**.

The number of **distinguishable permutations** of an n-set containing n distinguishable objects is

$$n!$$

If an n-set contains elements that are not distinguishable from one another, we must correct for overcounting. If an n-set contains n_1 objects of a first kind, n_2 objects of a second kind, etc., then the number of distinguishable permutations is

$$\frac{n!}{n_1!n_2!....n_k!}$$

Example 3: In how many ways can 6 people line up for a movie?

$\underline{6}$ $\underline{5}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{1}$ $= 6!$

$= \boxed{720}$

Example 4: Rewrite each expression $\boxed{\text{without the factorial (!) symbol.}}$

a) The first five factors of $(n-3)!$

$n-3-1$

$(n-3)(n-4)(n-5)(n-6)(n-7)$

b) $\dfrac{x!}{(x-1)!} = \dfrac{x\cancel{(x-1)}\cancel{(x-2)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{(x-1)}\cancel{(x-2)}(x-3)\cdots\cancel{2}\cdot\cancel{1}}$

$= \boxed{x}$

c) $\dfrac{(x+1)!}{(x-1)!} = \dfrac{(x+1)(x)\cancel{(x-1)}\cdots\cancel{3}\cancel{2}\cancel{1}}{\cancel{(x-1)}(x-2)\cdots\cancel{3}\cancel{2}\cancel{1}}$

$= \boxed{(x+1)(x)}$

Example 5: How many **distinguishable permutations** can be formed from the letters

a) H, U, S, K, Y

$\boxed{5! = 120}$

b) H, I, L, L, C, R, E, S, T

$\dfrac{9!}{2!} = \boxed{181{,}440}$

c) M, I, S, S, I, S, S, I, P, P, I

$\dfrac{11!}{\underset{\text{I's}}{4!}\,\underset{\text{S's}}{4!}\,\underset{\text{P's}}{2!}} = \boxed{34{,}650}$

⇒ Sometimes counting problems will come with **restrictions on the possible outcomes.**

Example 6: In how many ways can I arrange 3 different math books and 5 different history books on my bookshelf, if I require there to be a math book on both ends? START with Restrictions FIRST!!

$\underset{\text{3 math books to choose from}}{\underline{3}}\ \underbrace{\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}}_{\text{6 remaining books}}\ \underset{\text{2 math books to choose from}}{\underline{2}}$

$\boxed{6\cdot 6! = 4{,}320}$

Example 7: The Smith family has 4 sons and 3 daughters. In how many ways can they be seated in a row of 7 chairs such that all 3 girls sit next to each other?

treat as one object.

Imagine: G BBBB B G BBB BB G BB BBB G B BBBB G

ways to arrange $G, B_1, B_2, B_3, B_4 = 5!$

ways to arrange $G = 3!$

Possible seatings: $\boxed{5!\,3!} = 720$

⇒ In many permutation problems, we are interested in **using n objects to fill r blanks in order**, where $r < n$.

The number of permutations of n objects taken r at a time is denoted ${}_nP_r$ and is given by $${}_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \le r \le n$$

Example 8: Sixteen actors answer a casting call to try out for roles as dwarfs in a production of *Snow White and the Seven Dwarfs*. In how many different ways can the director cast the seven roles?

n = 16 actors
r = 7 roles

$${}_{16}P_7 = \underline{16}\ \underline{15}\ \underline{14}\ \underline{13}\ \underline{12}\ \underline{11}\ \underline{10} = \frac{16!}{(16-7)!} = \frac{16!}{9!}$$

$$= \boxed{57{,}657{,}600}$$

Example 9: In a lottery, 25 balls numbered 1 through 25 are placed in a bin. Four balls are drawn one at a time and their numbers are recorded. In how many ways can 4 balls be drawn, in order, from the bin, if:

a) Each ball is discarded after it is removed?

n = 25 lottery balls
r = 4 spots

$\underline{25}\ \underline{24}\ \underline{23}\ \underline{22}$

$${}_{25}P_4 = \frac{25!}{(25-4)!} = \frac{25!}{21!} = \boxed{303{,}600}$$

b) Each ball is placed back into the bin immediately after it is drawn? (Is this a permutation?) NO

$$\underline{25}\ \underline{25}\ \underline{25}\ \underline{25} = 25^4 = \boxed{390{,}625}$$

Example 10: Evaluate each expression without a calculator. Check your answer by using the ${}_nP_r$ function on your calculator.

a) ${}_6P_4$

$\frac{6!}{(6-4)!} = \frac{6!}{2!}$

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}$

$= \boxed{360}$

b) ${}_{11}P_3$

$\frac{11!}{8!}$

$\underline{11}\ \underline{10}\ \underline{9}$

$= \boxed{990}$

c) ${}_6P_6$

$\frac{6!}{0!} = 6!$

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$\boxed{= 6! = 720}$

d) ${}_{75}P_1$

$\frac{75!}{74!}$

$\underline{75}$

$= \boxed{75}$