Name: ______________________

Precalculus: UNIT 9
Lesson 2: Combinations

An ORDERED arrangement of n objects taken r at a time is called a **permutation**. Examples include:

- Seating arrangements
- Race results
- Electing officers
- PIN codes and lock combinations (as long as no object can be repeated)

An arrangement of n objects taken r at a time where ORDER DOES NOT MATTER is called a **combination**. Example include:

- Choosing committees
- Hands of cards
- Starting line-ups where positions are interchangeable

⇒ The number of combinations of n objects taken r at a time is called "n choose r" and is denoted by any of:

$$C(n,r) = {}_nC_r = \binom{n}{r}$$

Example 1: In how many ways can 3 people be chosen from a group of 8 people to form a committee?

$${}_8C_3 = \binom{8}{3} = \frac{8!}{3!\,5!} = \boxed{56}$$

$$\frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{3}\cdot \cancel{2}\cdot 1\ \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}$$

permutation

⇒ There is an easy formula for computing ${}_nC_r$:

> The number of combinations of n objects taken r at a time is given by
>
> $${}_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \le r \le n$$

Example 2: You shuffle a 52-deck several times and then deal yourself a hand of 5 cards. How many possible 5-card hands are there?

$${}_{52}C_5 = \binom{52}{5} = \frac{52!}{5!\,47!} = \boxed{2{,}598{,}960}$$

Example 3: You are in charge of forming an 8-person Junior Prom Committee. The Junior Class has 45 boys and 35 girls in it. How many distinct committees can be formed if:

a) there are no restrictions other than there must be 8 juniors on the committee?

$$_{80}C_8 = \binom{80}{8} = \frac{80!}{8!72!} = \boxed{2.89E10 = 28{,}900{,}000{,}000}$$

b) there must be 4 girls $\boxed{\text{and}}$ 4 boys on the committee?

$$\binom{35}{4}\cdot\binom{45}{4} = \frac{35!}{4!31!}\cdot\frac{45!}{4!41!} = 52{,}360 \cdot 148{,}995$$
$$= \boxed{7{,}801{,}378{,}200}$$

c) Sarah Dreadful cannot be on the committee?

$$\binom{79}{8} = \frac{79!}{8!71!} = \boxed{2.609E10}$$

Example 4: Coach Grunt is preparing the 5-person starting lineup for his basketball team. There are 12 players on the team and two of them, Ace and Zeppo, are league All-Stars and will definitely be in the starting line-up. How many different starting line-ups are possible (assuming every player can play every position)?

$$\binom{10}{3}$$

← players other than Ace and Zeppo

positions available for players other than Ace and Zeppo.

$$\frac{10!}{3!7!} = \boxed{120}$$

Example 5: Armando's Pizzeria offers patrons a choice of up to 10 different toppings.

a) How many different pizzas can be ordered if we choose 3 $\boxed{\text{or}}$ fewer toppings?

$$\binom{10}{3}+\binom{10}{2}+\binom{10}{1} = 120+45+10$$
$$= \boxed{175}$$

b) How many different pizzas can be ordered if we choose *any* number of toppings (0 through 10)?

$$\binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\binom{10}{3}+\ldots+\binom{10}{9}+\binom{10}{10}$$
$$1+10+45+120+210+252+210+120+45+10+1$$
$$= \boxed{1024}$$

$$\binom{4}{3} \stackrel{?}{=} \binom{4}{1}$$

$$\frac{4!}{3!\,1!} \qquad \frac{4!}{1!\,3!}$$

ABCD

ABC	A
ABD	B
ACD	C
BCD	D

$$\boxed{\binom{10}{7} = \binom{10}{3}}$$