Precalculus – Quarter 1 Final Review Topics

Prerequisites Chapter

- Write inequalities in interval notation.

1. Write $x < 3$ in interval notation.

$(-\infty, 3]$

2. Write [3, 5] as an inequality.

$3 \leq x \leq 5$

- Simplify expressions using properties of exponents

3. $\left(\dfrac{3xy^{-2}}{2x^3y}\right)^3 = \dfrac{3^3x^3y^{-6}}{2^3x^9y^3}$

$= \dfrac{27}{8x^6y^9}$

4. $(2a^2b^3)^4(3a^3b)$

$(2^4a^8b^{12})(3a^3b)$

$= (16a^8b^{12})(3a^3b)$

$= 48a^{11}b^{13}$

- Solve linear equations

5. $3x + 2(4x - 3) = 5x$

$3x + 8x - 6 = 5x$

$11x - 6 = 5x$

$-11x \quad -11x$

$-6 = -6x$

$\dfrac{-6}{-6} = \dfrac{-6x}{-6}$

$x = 1$

6. $\dfrac{4x-5}{3} + \dfrac{2x+7}{2} = 4$ LCD: 6

$\dfrac{2(4x-5)}{6} + \dfrac{3(2x+7)}{6} = \dfrac{6(4)}{6}$

$= \dfrac{8x - 10 + 6x + 21}{6} = \dfrac{24}{6}$

$= 14x + 11 = 24$

$-11 \quad -11$

$\dfrac{14x}{14} = \dfrac{13}{14}$

$x = \dfrac{13}{14}$

- Solve linear inequalities

7. $5 \geq \dfrac{2-3x}{2} > 8$ ·2

$10 \geq 2 - 3x > 16$

$-2 \quad -2 \quad -2$

$\dfrac{8}{-3} \geq \dfrac{-3x}{-3} > \dfrac{14}{-3}$

$-\dfrac{8}{3} \leq x < -\dfrac{14}{3}$

- Write the equation of a line when given: $y = mx + b$

A point on the line and the slope

8. (2, –4), slope = ½

$(x, y) \quad m$

$-4 = \frac{1}{2}(2) + b$

$-4 = 1 + b$

$-1 \quad -1$

$-5 = b$

$y = \frac{1}{2}x - 5$

Two points on the line.

9. (–4, 7) and (–1, 10)

$m = \dfrac{10-7}{-1+(-4)} = \dfrac{3}{3} = 1$

pick a point: $(-1, 10) \rightarrow (x, y)$

$10 = 1(-1) + b$

$10 = -1 + b$

$+1 \quad +1$

$11 = b$

$y = 1x + 11$

10. $(3, 5)$ and perpendicular to $y = \frac{1}{4}x - 6$ $m = \frac{1}{4}$

(x,y)

$m = -4$

$5 = -4(3) + b$

$5 = -12 + b$

$+12 \quad +12$

$17 = b$

$y = -4x + 17$

- Solve quadratics by
  - Factoring, taking square roots, and using the quadratic formula

11. $x^2 + 10x + 24 = 0$

24	10
6, 4	10

$(x+6)(x+4) = 0$

$x = -6$ $x = -4$

12. $12x^2 - 8x = 0$

$4x(3x - 2) = 0$

$\frac{4x}{4} = \frac{0}{4}$ $3x - 2 = 0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$x = 0$ $x = \frac{2}{3}$

13. $\sqrt{(2x-3)^2} = \sqrt{81}$

$2x - 3 = \pm 9$

$2x - 3 = 9$ $2x - 3 = -9$

$+3 \quad +3$ $+3 \quad +3$

$\frac{2x}{2} = \frac{12}{2}$ $\frac{2x}{2} = \frac{-6}{2}$

$x = 6$ $x = -3$

14. $3x^2 - 4x + 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{16 - 4(3)(5)}}{2(3)} = \frac{4 \pm \sqrt{16 - 60}}{6}$

$x = \frac{4 \pm \sqrt{-44}}{6} = \frac{4 \pm 2i\sqrt{11}}{6}$

$x = \frac{2 \pm i\sqrt{11}}{3}$

- Solve absolute value equations and inequalities

15. $|3 - 4x| = 32$

$3 - 4x = 32$ $-(3 - 4x) = 32$

$-3 \quad -3$ $-3 + 4x = 32$

$+3 \quad +3$

$\frac{-4x}{-4} = \frac{29}{-4}$ $\frac{4x}{4} = \frac{35}{4}$

$x = \frac{-29}{4}$ $x = \frac{35}{4}$

16. $|x - 5| > 8$

$x - 5 > 8$ $-(x - 5) > 8$

$+5 \quad +5$ $-x + 5 > 8$

$-5 \quad -5$

$\frac{-x}{-1} > \frac{3}{-1}$

$x > 13$ $x < -3$

- Solve quadratic inequalities.

17. $x^2 + 8x + 12 \geq 0$

12	8
6, 2	8

$(x+6)(x+2) \geq 0$

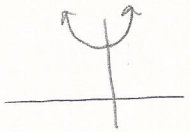
$x = -6 \quad x = -2$

$(-\infty, -6] \cup [-2, \infty)$

18. $x^2 + 3 < 0$

$x^2 < -3$

$x < \pm\sqrt{-3}$ ← no real solution

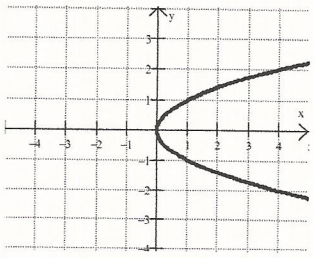


NO solution

Functions Chapter

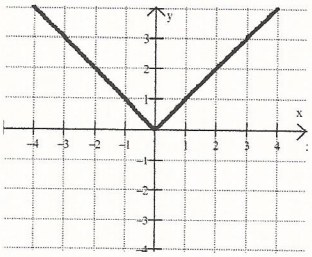
- Determine if a relation is a function.

1.



Not a function, fails the vertical line test.

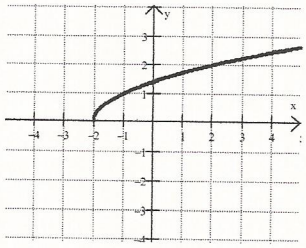
2.



Is a function, passes the vertical line test.

- Find the domain and range of a function when
  - Given the graph of the function or given the equation of the function

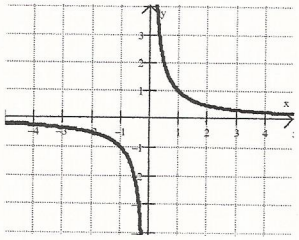
3.



D: $[-2, \infty)$

R: $[0, \infty)$

4.



D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

5. $f(x) = x^2 + 6x - 2$

D: $(-\infty, \infty)$

6. $f(x) = \dfrac{x-3}{x+5}$

D: $(-\infty, -5) \cup (-5, \infty)$

7. $f(x) = \sqrt{2x-3}$

$2x - 3 \geq 0$

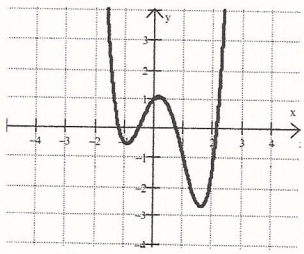
$2x \geq 3$

$x \geq 3/2$

D: $[\frac{3}{2}, \infty)$

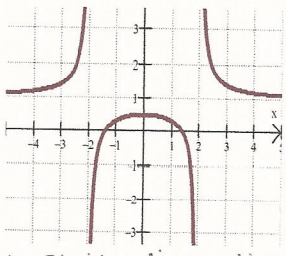
- Identify the following characteristics of a function
  - Continuity. If a function is discontinuous determine if it is a jump, removable or infinite discontinuity.
  - Boundedness
  - Increasing/decreasing
  - Extrema. Absolute and local max and mins.
  - Odd, even or neither
  - Asymptotes
  - End behavior

8.



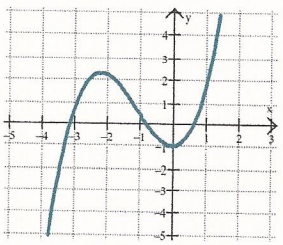
- continuous
- bounded below
- Increasing: $(-1, 0) \cup (1.5, \infty)$
  Decreasing: $(-\infty, -1) \cup (0, 1.5)$
- local min: -1.5 when $x = -1$
  local max: 1 when $x = 0$
  absolute min: -2.5 when $x = 1.5$
- Neither
- None
- $\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = \infty$

9.



- infinite discontinuity
- not bounded
- Increasing: $(-\infty, -2) \cup (-2, 0)$
  Decreasing: $(0, 2) \cup (2, \infty)$
- None
- even
- asymptotes: $y = 1$, $x = \pm 2$
- $\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to \infty} f(x) = 1$

10.



- continuous
- not bounded
- increasing: $(-\infty, -2) \cup (0, \infty)$
  Decreasing: $(-2, 0)$
- local max: 2 when $x = -2$
  local min: -1 when $x = 0$
- neither
- none
- $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

- Combine functions algebraically ($f+g, f-g, fg, f/g$ and $f(g(x))$) and state the new domain.

$f(x)=x^2-3x$ $\quad g(x)=\sqrt{x-3}$ $\quad h(x)=4x-1$

11. $f+h$

$(x^2-3x)+(4x-1)$

$=x^2+x-1$ $\quad D:(-\infty,\infty)$

12. h/g

$\frac{4x-1}{\sqrt{x-3}}$ $\quad D: x\geq 3$

13. $f(h(x))$

$(4x-1)^2-3(4x-1)$

$=16x^2-8x+1-12x+3$

$=16x^2-20x+4$ $\quad D:(-\infty,\infty)$

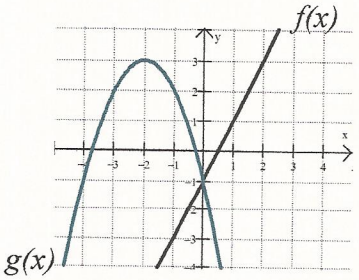
14. g/h

$\frac{\sqrt{x-3}}{4x-1}$ $\quad D: x\neq \frac{1}{4}$

$(-\infty,\frac{1}{4})\cup(\frac{1}{4},\infty)$

- Evaluate functions algebraically and graphically.

$f(x)=3x-1$ $\quad g(x)=x^2-5$



15. $f(2)=3(2)-1$

$=6-1$

$=5$

16. $f(g(3))$

$g(3)=(3)^2-5=9-5=4$

$f(4)=3(4)-1=12-1=11$

17. $g(-3)=(-3)^2-5$

$=9-5=4$

18. $f(2)$

$=3(2)-1$

$=6-1=5$

19. $f(g(-4))$ $\quad g(-4)=(-4)^2-5=16-5=9$

$f(9)=3(9)-1=27-1=26$

- Find (x, y) on a parametric function for a given value of t. Then eliminate the parameter, write y as a function of x.

20. $x=t-4$, $y=5x^2-7$, $t=3$

$x=3-4=-1$

$y=5(3)^2-7=5(9)-7=45-7=38$ $\quad (-1,38)$

- Find the inverse of a function.

21. $y=\sqrt{x-5}+1$

$x=\sqrt{y-5}+1$

$-1 \quad -1$

$x-1=\sqrt{y-5}$

$(x-1)^2=y-5$

$+5 \quad +5$

$y=(x-1)^2+5$

22. $y=\frac{x+5}{3-2x}$ $\quad x=\frac{y+5}{3-2y}(3-2y)$

$x(3-2y)=y+5$

$3x-2xy=y-5$

$-2xy+y=-3x-5$

$y(-2x+1)=-3x-5$

$y=\frac{-3x-5}{-2x+1}$

Polynomials

- Determine if the function is a polynomial, if it is, determine the degree and leading coefficient

23. $f(x)=x^{\frac{2}{3}}+7x+1$ No, not a function. (Has a rational exponent)

24. $f(x)=10-3x^2$ yes

degree: 2

leading coefficient: -3

- Write the quadratic in vertex form.

25. $f(x)=10-3x^2$

$f(x)=-3x^2+10$

(vertex: (0, 10))

26. $y=x^2+6x-3$

$y=(x^2+6x+\underline{9})-3-\underline{9}$

$\frac{6}{2}=3 \quad (3)^2=9$

$y=(x^2+6x+9)-12$

$y=(x+3)^2-12$ $\quad$ (vertex: (-3, -12))

27. $y=2x^2+8x+5$

$y=(2x^2+8x+\_\_)+5-\_\_$

$y=2(x^2+4x+\underline{4})+5-(2)\underline{4}$

$y=2(x+2)^2+5-8$

$\boxed{y=2(x+2)^2-3}$

28. $y=-3x^2+12x+1$

$y=-3(x^2-4x+\_\_)+1-(-3)\_\_$

$y=-3(x^2-4x+4)+1-(-3)(4)$

$\boxed{y=-3(x-2)^2+13}$

- Write the equation of a quadratic when given a vertex and a point. $y=a(x-h)^2+k$

29. vertex: (2, 4), point: (0, -3)

(h, k) (x, y)

$-3=a(0-2)^2+4$

$-3=4a+4$ $\quad -7=4a$

$-4 \quad -4 \quad a=-\frac{7}{4}$

$\boxed{y=-\frac{7}{4}(x-2)^2+4}$

30. vertex: (-3, 2), point: (1, 5)

$5=a(1-(-3))^2+2$

$\frac{5}{-2}=\frac{16a}{-2}+\frac{2}{-2}$ $\quad a=\frac{3}{16}$

$3=16a$

$\boxed{y=\frac{3}{16}(x+3)^2+2}$

- Write an equation for the linear function f satisfying the given conditions.

31. $f(1)=3$, ~~$f(1)=3$~~ $f(-3)=7$

(1, 3) (-3, 7)

$m=\frac{7-3}{-3-1}=\frac{4}{-4}=-1$

$y=mx+b$

$3=-1(1)+b$

$3=-1+b$

$b=4$

$\boxed{y=-1x+4}$

- Find the zeros of a polynomial.

32. $f(x)=x^2+7x+10$

$0=(x+2)(x+5)$

$x+2=0$ $\quad x+5=0$

$\boxed{x=-2}$ $\quad \boxed{x=-5}$

33. $f(x)=5x^3-45x$

$0=5x(x^2-9)$

$0=5x(x-3)(x+3)$

$5x=0$ $\quad x-3=0$ $\quad x+3=0$

$\boxed{x=0}$ $\quad \boxed{x=3}$ $\quad \boxed{x=-3}$

34. $f(x)=2x^3-5x^2-12x$

$0=2x(x^2-5x-6)$

$0=2x(x-6)(x+1)$

$2x=0$ $\quad x-6=0$ $\quad x+1=0$

$\boxed{x=0}$ $\quad \boxed{x=6}$ $\quad \boxed{x=-1}$

- Find a polynomial with the given zeros.

35. zeros: 0, -3, 1

$f(x)=(x+0)(x+3)(x-1)$

$=x(x+3)(x-1)$

$=x(x^2-x+3x-3)$

$=x(x^2+2x-3)$

$\boxed{f(x)=x^3+2x^2-3x}$

36. zeros: -1, -2, 2

$f(x)=(x+1)(x+2)(x-2)$

$=(x+1)(x^2-4)$

$=x^3-4x+x^2-4$

$\boxed{f(x)=x^3+x^2-4x-4}$

- Story Problems.

37. How much 15% solution and how much 25% solution should be mixed together to make 50 gallons of 20% solution.

$.15x+.25(50-x)=.2(50)$

$.15x+12.5-.25x=10$

$-.1x=10-12.5$

$-.1x=-2.5$

$x=25$

25 gallons of 15%
25 gallons of 25%.

38. Steve invests $15,000, part at 4.5% annual interest and the balance at 7.3% annual interest. How much is invested at each rate if Steve receives a 1 year interest payment of $871.

$x + y = 15{,}000$

$.045x + .073y = 871$

$.045x + .073(15000 - x) = 871$

$x = 8000$

$8,000 at 4.5%
$7,000 at 7.3%

Calculator Portion

- Find the x-intercepts and any max or mins.

1. $f(x) = x^3 - 3x^2 + 2$

$x = 1$
$x = -0.732$
$x = 2.73$

maximum value: 2 when $x = 0$

minimum value: -2 when $x = 2$

2. $f(x) = -0.2x^5 - 2x^2 + 0.5x + 1$

$x = -4.69$
$x = -0.59$
$x = 0.84$

max value: 1.031 when $x = 0.125$

min value: -14.76 when $x = -3.46$

- Solve inequalities

3. $3x^4 < 2x^2$

$(-0.816, 0) \cup (0, 0.816)$

4. $x^3 + 5x^2 - x - 5 > 0$

$(-5, 1) \cup (1, \infty)$

5. $x^2 - 4 < 0$

$(-2, 2)$

- Find the domain and range.

6. $y = \sqrt{5 - x}$

D: $(-\infty, 5]$

R: $[0, \infty)$

7. $y = (x+3)^2 - 7$

D: $(-\infty, \infty)$

R: $[-7, \infty)$

- Determine if a function is even or odd by graphing.

8. $f(x) = |x^2 + 7x|$

neither

9. $f(x) = \sqrt{x^2 - 1}$

even