2.3: Continuity

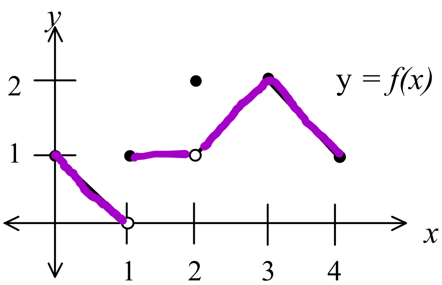
Continuity at a Point

Interior point: A function *f(x)* is continuous at an interior point *c* of its domain if

$$\lim_{x \to c} f(x) = f(c)$$

Endpoint: A function *f(x)* is continuous at left endpoint a or its right endpoint *b* of its domain if

Example 1: Finding Continuity Graphically



a) Where is f(x) continuous?

$[0,1) \cup (1,2) \cup (2,4]$

b) Not Continuous?

$x = 1$ $x = 2$

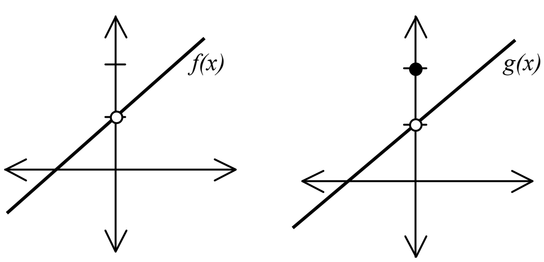
c) Compare this to the limits at these values.

$\lim_{x \to 1} f(x) = \text{DNE}$ $\lim_{x \to 2} f(x) = 1$

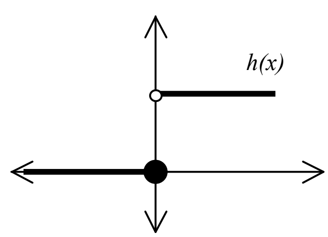
$f(1) = 1$ $f(2) = 2$

Types of Discontinuity

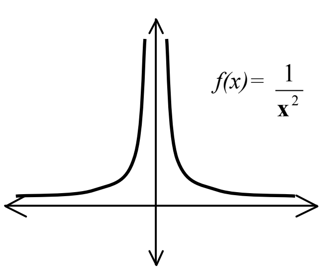
Removable Discontinuity:



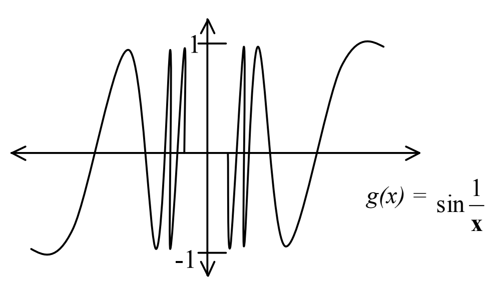
Jump discontinuity:



Infinite discontinuity:



Oscillating discontinuity:



Exploration 1: Let $f(x) = \dfrac{x^3 - 7x - 6}{x^2 - 9}$

1. Factor the denominator. What is the domain of *f*?

$x^2 - 9 = (x+3)(x-3)$ D: $x \neq \pm 3$ $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

2. Investigate the graph of *f* around x = 3 to see that *f* has a removable discontinuity at x = 3.

3. How should *f* be defined at x = 3 to remove the discontinuity? Use zoom-in and tables as necessary.

$\lim_{x\to 3} \dfrac{x^3-7x-6}{x^2-9} = \lim_{x\to 3} \dfrac{\cancel{(x-3)}(x^2+3x+2)}{\cancel{(x-3)}(x+3)} = \lim_{x\to 3} \dfrac{x^2+3x+2}{x+3} = \dfrac{20}{6} = \boxed{\dfrac{10}{3}}$

4. Show that (x – 3) is a factor of the numerator of *f*, and remove all common factors. Now compute the limit as x → 3 of the reduced form for *f*.

3	1	0	-7	-6
	↓	3	9	6
	1	3	2	0

5. Write the *extended function* so that it is continuous at x = 3.

$$g(x) = \begin{cases} \dfrac{x^3-7x-6}{x^2-9} & x \neq 3 \\ \dfrac{10}{3} & x = 3 \end{cases}$$

**The function *g* is the <u>continuous extension</u> of the original function *f* to include x = 3.

Example 1: Determine if each function is continuous without graphing.

a) $f(x) = \begin{cases} 3x+2 & x<0 \\ x-4 & x \geq 0 \end{cases}$

$\lim_{x\to 0^+} f(x) = -4$

$\lim_{x\to 0^-} f(x) = 2$

NOT CONTINUOUS

b) $f(x) = \begin{cases} x^2-3 & x<-1 \\ x-1 & x \geq -1 \end{cases}$

$\lim_{x\to -1^+} f(x) = -2$

$\lim_{x\to -1^-} f(x) = -2$

$f(-1) = -2$

CONTINUOUS

c) $g(x) = \begin{cases} 3-x & x<2 \\ 2 & x=2 \\ \dfrac{x}{2} & x>2 \end{cases}$

$\lim_{x\to 2^+} f(x) = 1$

$\lim_{x\to 2^-} f(x) = 1$

$f(2) = 2$

NOT continuous

Example 2: Determine the value for *a* so that the function is continuous.

a) $f(x) = \begin{cases} 2x+3 & x \leq 2 \\ ax+1 & x>2 \end{cases}$

$\lim_{x\to 2^-} f(x) = 4+3 = 7$

$\lim_{x\to 2^+} f(x) = a(2)+1 = 7$

$-1 \quad -1$

$2a = 6$

$\boxed{a=3}$

b) $f(x) = \begin{cases} x^2+x+a & x<1 \\ x^3 & x \geq 1 \end{cases}$

$\lim_{x\to 1^+} f(x) = 1$

$\lim_{x\to 1^-} f(x) = (1)^2+1+a = 1$

$1+1+a=1$

$2+a=1$

$\boxed{a=-1}$

$$f(x)=\frac{\cancel{x}\cancel{(x-1)^2}\cancel{(x-2)}(x-3)^3\cancel{(x-4)^2}}{\cancel{x}\cancel{(x-1)^2}(x-2)^{\cancel{2}}\cancel{(x-3)^2}\cancel{(x-4)^2}}$$

a. $x=0$

b. $x=1$

c. $x=2$

d. $x=3$

e. $x=4$

<u>NOT</u> removable

$$f(x)=\frac{x-3}{x-2}$$

$\neq 0$

$x \neq 2$