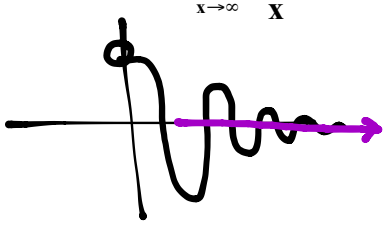
2.2: Limits Involving Infinity

Horizontal Asymptote:	Vertical Asymptote:
$\lim_{x\to\pm\infty} f(x) = \pm a$	$\lim_{x\to\pm a} f(x) = \pm\infty$

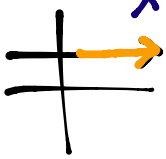
Example 1: Finding a Limit as x Approaches Infinity

a) $\lim_{x\to\infty} \frac{\sin x}{x} =$ 0



b) $\lim_{x\to\infty} \frac{3x+\sin x}{x} =$ $\frac{3x}{x} + \frac{\sin x}{x}$

$\lim_{x\to\infty} \frac{3x}{x} + \lim_{x\to\infty} \frac{\sin x}{x}$



$\lim_{x\to\infty} 3 + \lim_{x\to\infty} \frac{\sin x}{x}$

$= 3 + 0 = \boxed{3}$

Example 2: Find the vertical or horizontal asymptotes of each function.

a) $f(x) = \frac{1}{x+2}$ $\frac{1}{x}$

H.A.: $y = 0$

V.A.: $x = -2$

b) $g(x) = \frac{2x^2-11}{x^2+9}$ $x^2+9=0$, $x^2=-9$, $x=\pm 3i$

H.A.: $y = 2$

V.A.: NONE

c) $h(x) = \frac{x^2+2x-3}{x^2+5x-6} = \frac{(x+3)(x-1)}{(x+6)(x-1)}$

H.A.: $y = 1$

V.A: $x = -6$

★ Hole at $x = 1$ ★

Example 3: Finding End Behavior Models

Find the end behavior for

a) $f(x) = \frac{3x^4+x^3-5x^2+4}{2x^2-3x+5}$

$\lim_{x\to\infty} \frac{3x^4}{2x^2} = \lim_{x\to\infty} \frac{3x^2}{2} = \infty$

$\lim_{x\to-\infty} \frac{3x^4}{2x^2} = \lim_{x\to-\infty} \frac{3x^2}{2} = \infty$

b) $f(x) = \frac{2x^2-4x+3}{5x^2+8}$

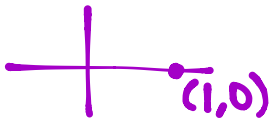
$\lim_{x\to\infty} \frac{2x^2}{5x^2} = \lim_{x\to\infty} \frac{2}{5} = \boxed{\frac{2}{5}}$

$\lim_{x\to-\infty} \frac{2x^2}{5x^2} = \lim_{x\to-\infty} \frac{2}{5} = \frac{2}{5}$

Example 4: Using Substitution.

a) $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) =$ $\frac{1}{\infty} = 0$

$\sin(0) = \boxed{0}$



b) $\lim_{x\to\infty} \left(2 + \frac{5}{x}\right)$

$\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{5}{x}$

$2 + 0 = \boxed{2}$