1.4: Building Functions from Functions

Example 1: Evaluating the functions algebraically.

$f(x) = 3x^2 - 5, \quad g(x) = 5 - 2x$

a) $f(3) =$

$= 3(3)^2 - 5$
$= 3(9) - 5$
$= 27 - 5$
$= \boxed{22}$

b) $g(-1) =$

$= 5 - 2(-1)$
$= 5 + 2$
$= \boxed{7}$

c) $g(f(2)) =$

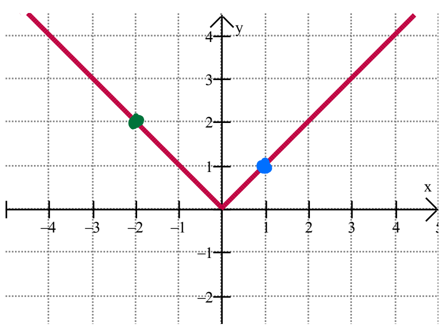
$f(2) = 3(2)^2 - 5$
$= 3(4) - 5$
$= 12 - 5$
$= 7$
$g(7) = 5 - 2(7)$
$= 5 - 14$
$= -9$
$\boxed{g(f(2)) = -9}$

d) $f(g(-1)) =$

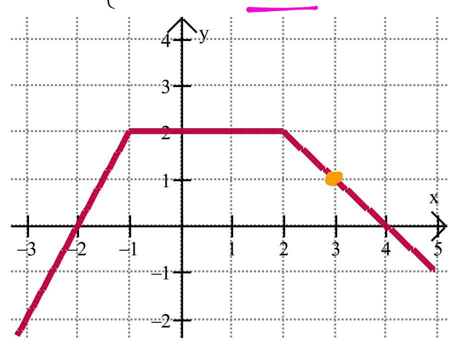
$g(-1) = 5 - 2(-1)$
$= 5 + 2$
$= 7$
$f(7) = 3(7)^2 - 5$
$= 3(49) - 5$
$= 147 - 5$
$= 142$
$\boxed{f(g(-1)) = 142}$

Example 2: Evaluating the function graphically.

$f(x) = |x|$



$$g(x) = \begin{cases} 2x + 4 & x < -1 \\ 2 & -1 \le x \le 2 \\ -x + 4 & x > 2 \end{cases}$$



a) $f(-2) = 2$

b) $f(1) = 1$

c) $g(3) = 1$

$-x + 4$
$-3 + 4 = 1$

d) $g(1) = 2$

2

e) $g(5) = -1$

$-x + 4$
$-5 + 4 = -1$

Combining Functions Algebraically

Example 3: Use $f(x)$ and $g(x)$ to find the formulas for each new functions below. Give the domain of each.

$f(x)=x^2-4$ $\quad$ $g(x)=2x+3$ $\quad$ $h(x)=\sqrt{3x+1}$

a) $f-g$

$(x^2-4)-(2x+3)$

$=x^2-2x-7$

D: $(-\infty,\infty)$

b) $f\cdot g$

$(x^2-4)(2x+3)$

$=2x^3+3x^2-8x-12$

D: $(-\infty,\infty)$

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c) $\frac{g}{h}=\frac{2x+3}{\sqrt{3x+1}}$

Domain: $3x+1>0$

$3x>-1$

$x>-\frac{1}{3}$

D: $x>-\frac{1}{3}$

D: $(-\frac{1}{3},\infty)$

d) $\frac{h}{f}=\frac{\sqrt{3x+1}}{x^2-4}$

Domain:

$x^2-4\neq 0$

$(x-2)(x+2)\neq 0$

$x\neq 2$ $\quad x\neq -2$

$3x+1\geq 0$

$3x\geq -1$

$x\geq -\frac{1}{3}$

-2 $\quad -\frac{1}{3}$ $\quad 2$

D: $[-\frac{1}{3},2)\cup(2,\infty)$

D: $x\geq -\frac{1}{3}, x\neq 2$

Composition of Functions

The domain of $(f\circ g)(x)=f(g(x))$

Domain: · must include the domain of the inside function $(g(x))$ as well as the final function $(f(g(x)))$.

e) $f(h(x))$

Domain of $h(x)$: $x\geq -\frac{1}{3}$

$f(\sqrt{3x+1})=(\sqrt{3x+1})^2-4$

$=3x+1-4$

$=3x-3$

$f(h(x))=3x-3$

Domain of $f(h(x))$: $(-\infty,\infty)$

D: $x\geq -\frac{1}{3}$

f) $h(g(x))$

Domain of $g(x)$: $(-\infty,\infty)$

$h(2x+3)=\sqrt{3(2x+3)+1}$

$=\sqrt{6x+9+1}$

$h(g(x))=\sqrt{6x+10}$

Domain of $h(g(x))$:

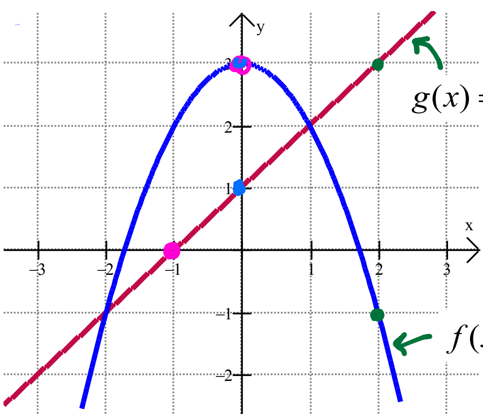
$6x+10\geq 0$

$6x\geq -10$

$x\geq -10/6$ $\quad x\geq -\frac{5}{3}$

D: $[-\frac{5}{3},\infty)$

Example 4: Use the graphs of $f(x)$ and $g(x)$ to evaluate the functions below.



a) $h(x)=f(x)+g(x)$, find $h(2)$.

$h(2)=f(2)+g(2)$

$=-1+3$

$=2$

$h(2)=2$

b) $r(x)=f(g(x))$, find $r(-1)$

$r(-1)=f(g(-1))$

$g(-1)=0$

$f(0)=3$

$r(-1)=3$

c) $m(x)=f(x)-g(x)$, find $m(0)$

$m(0)=f(0)-g(0)$

$=3-1$

$=2$

$m(0)=2$

Decomposing Functions

Example 5: For each function h, find functions f and g such that $h(x)=f(g(x))$.

a) $h(x)=(x+1)^2-3(x+1)+4$

$g(x)=x+1$

$f(x)=x^2-3x+4$

b) $h(x)=\sqrt{x^3+4}$

$g(x)=x^3+4$

$f(x)=\sqrt{x}$