1.7: Modeling with Functions

Example 1: Write the area A of a circle as a function of its

a) radius r. $A=\pi r^2$

b) diameter d. $A=\pi\left(\frac{d}{2}\right)^2$

$\frac{d}{2}=\frac{2r}{2}$

$r=\frac{d}{2}$

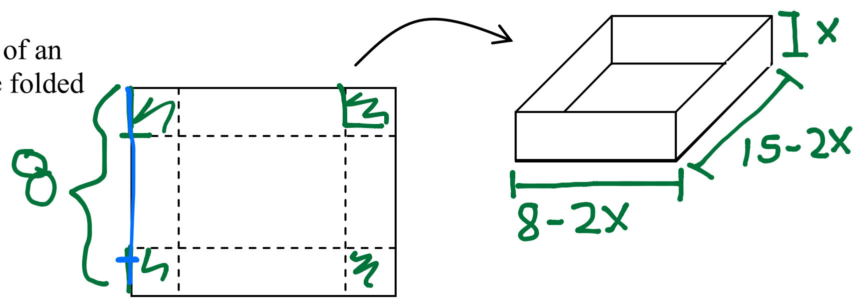
c) circumference C. $A=\pi\left(\frac{C}{2\pi}\right)^2$

$\frac{C}{2\pi}=\frac{2\pi r}{2\pi}$

$r=\frac{C}{2\pi}$

Example 2: **A Maximum Value Problem**

A square of side x inches is cut out of each corner of an 8 in. by 15 in. piece of cardboard and the sides are folded up to form an open-topped box.



a) Write the volume V of the box as a function x.

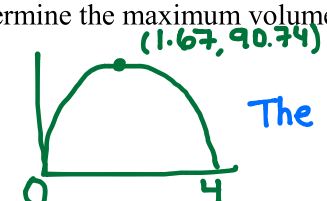
$V=l\cdot w\cdot h$

$\boxed{V=(8-2x)(15-2x)(x)}$

b) Find the domain of V as a function of x. $x\to$ how much we cut out.

$D:(0,4)$

c) Graph V as a function of x over the domain bound in part (b) and use the maximum finder on your calculator to determine the maximum volume such a box can hold.



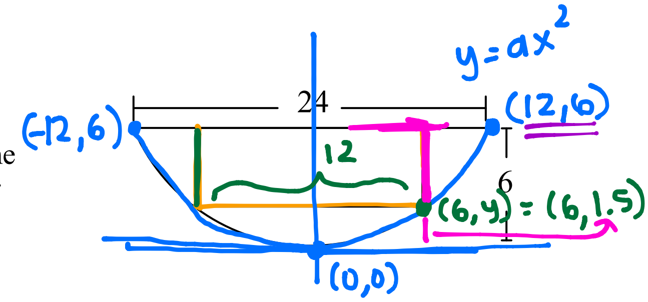
The maximum volume is 90.74 in^3 when x is 1.67 in.

d) How big should the cut-out squares be in order to produce the box of maximum volume?

Each square should have sides of 5/3 or 1 2/3 in.

Example 3: **Protecting an Antenna**

A small satellite dish is packaged with a cardboard cylinder for protection. The parabolic dish is 24 in. diameter and 6 in. deep, and the diameter of the cardboard cylinder is 12 in. How tall must the cylinder be to fit in the middle of the dish and be flush with the top of the dish?



$y = ax^2 \quad (12,6)$

$6 = a(12)^2$

$\frac{6}{144} = \frac{144a}{144}$

$a = \frac{6}{144} = \frac{1}{24}$

$y = \frac{1}{24}x^2$

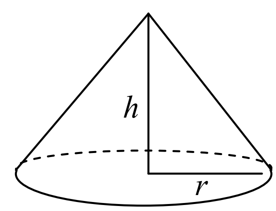
$(6, y)$

$y = \frac{1}{24}(6)^2 = \frac{36}{24} = 1.5$

$6 - 1.5 = \boxed{4.5\text{ in}}$

Example 4: **Sand Pile**

Grain is leaking through a hole in a storage bin at a constant rate of 8 cubic inches per minute. The grain forms a cone-shaped pile on the ground below. As it grows, the height of the cone always remains equal to its radius. If the cone is one foot tall now, how tall will it be in one hour?



Volume of the cone: $V = \frac{1}{3}\pi r^2 h$

$r = h$

$V = \frac{1}{3}\pi h^3$

$60\text{min}\left(8\,\text{in}^3/\text{min}\right) = 480\,\text{in}^3$

In 1 hour the volume will have grown

$h = 12 : V = 576\pi\ \text{in}^3$

The total volume is $(576\pi + 480)\,\text{in}^3$

$V = 576\pi + 480$

$3 \cdot \frac{1}{3}\pi h^3 = (576\pi + 480)3$

$\frac{\pi h^3}{\pi} = \frac{3(576\pi + 480)}{\pi}$

$h = \sqrt[3]{\frac{3(576\pi + 480)}{\pi}}$

$\boxed{h \approx 12.98 \text{ inches}}$