

P.7: Solving Inequalities Algebraically and Graphically

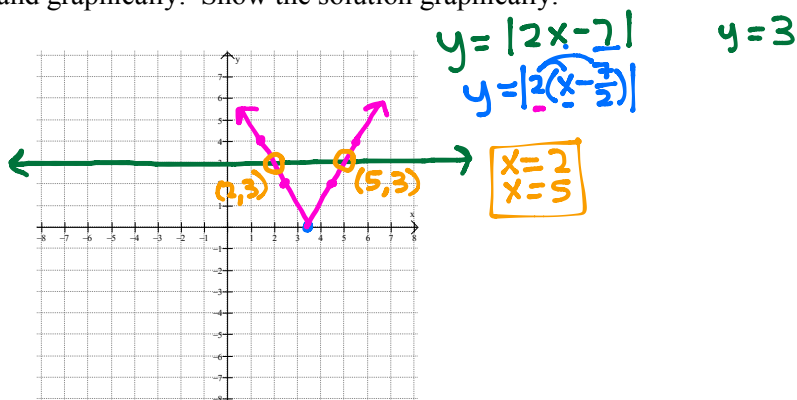
Absolute value function:

Example 1: Solve the absolute value equation by hand and graphically. Show the solution graphically.

a) $|2x - 7| = 3$

$$\begin{array}{r} 2x - 7 = 3 \\ +7 \quad +7 \\ \hline 2x = 10 \\ \frac{2x}{2} = \frac{10}{2} \\ \boxed{x = 5} \end{array}$$

$$\begin{array}{r} -(2x - 7) = 3 \\ -2x + 7 = 3 \\ \quad -7 \quad -7 \\ \hline -2x = -4 \\ \frac{-2x}{-2} = \frac{-4}{-2} \\ \boxed{x = 2} \end{array}$$



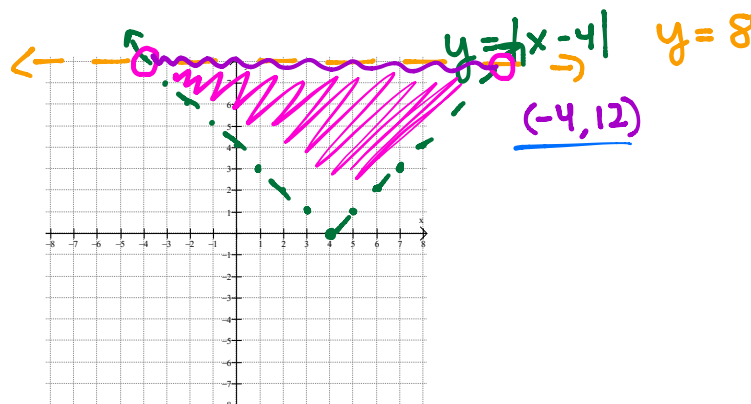
Example 2: Solve the absolute value inequality by hand and graphically. Show the solution graphically.

a) $|x - 4| < 8$

$$\begin{array}{r} x - 4 < 8 \\ +4 \quad +4 \\ \hline x < 12 \end{array}$$

$$\begin{array}{r} -(x - 4) < 8 \\ -x + 4 < 8 \\ \quad -4 \quad -4 \\ \hline -x < 4 \\ \frac{-x}{-1} < \frac{4}{-1} \\ x > -4 \end{array}$$

$-4 < x < 12$



Rule for solving absolute value inequalities.

Example 3: Solve the inequality algebraically (by hand).

a) $|3x - 2| \geq 5$

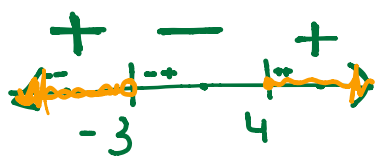
$$\begin{array}{r} 3x - 2 \geq 5 \\ +2 \quad +2 \\ \hline 3x \geq 7 \\ \frac{3x}{3} \geq \frac{7}{3} \\ x \geq \frac{7}{3} \end{array}$$

$$\begin{array}{r} -(3x - 2) \geq 5 \\ -3x + 2 \geq 5 \\ \quad -2 \quad -2 \\ \hline -3x \geq 3 \\ \frac{-3x}{-3} \geq \frac{3}{-3} \\ x \leq -1 \end{array}$$

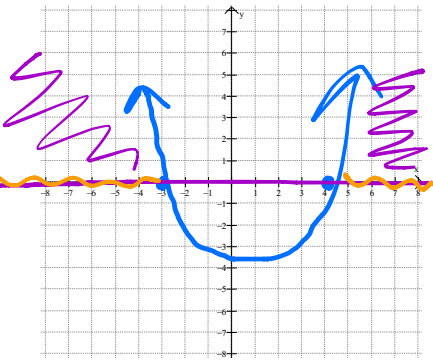
$x \leq -1$ or $x \geq \frac{7}{3}$
 $(-\infty, -1] \cup [\frac{7}{3}, \infty)$

Example 4: Solve the quadratic inequalities algebraically and graphically.

a) $x^2 - x - 12 > 0$ ✓ positive
 $(x-4)(x+3)$

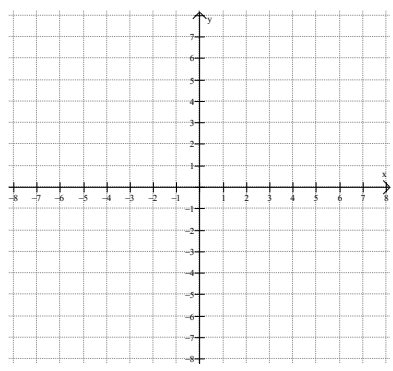
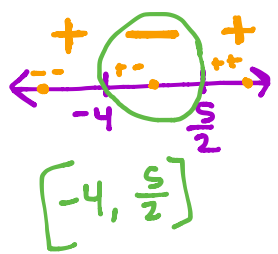


$(-\infty, -3) \cup (4, \infty)$



b) $2x^2 + 3x \leq 20$
 $\frac{-20 \quad -20}{2x^2 + 3x - 20 \leq 0}$

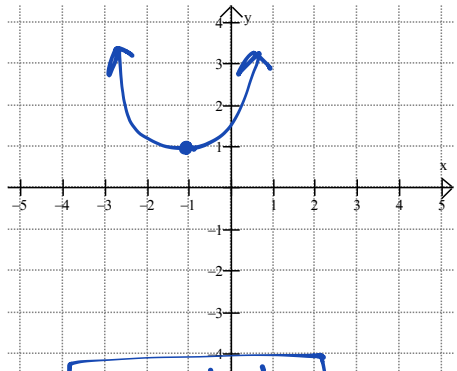
$\frac{-40 \quad 3}{8, -5} \quad 2x^2 + 8x - 5x - 20 \leq 0$
 $2x(x+4) - 5(x+4) \leq 0$
 $(x+4)(2x-5) \leq 0$
 $x = -4 \quad x = \frac{5}{2}$



Example 5: Solve the inequalities graphically.

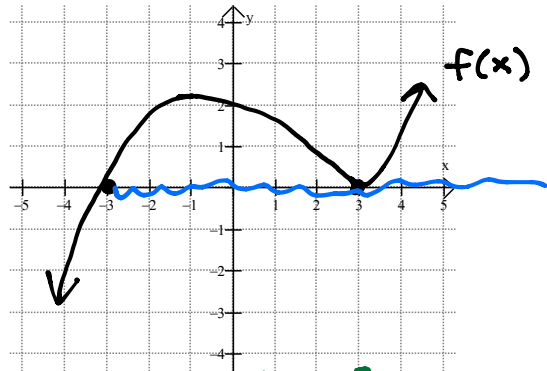
a) $x^2 + 2x + 2 < 0$
 $(x^2 + 2x + 1) + 2 - 1 < 0$
 $(x+1)^2 + 1 < 0$

b) ~~$x^2 + 2x + 2 > 0$~~



no solution

b) $x^2 + 2x + 2 > 0 \quad \mathbb{R}$



a) when is $f(x) > 0$?
 $(-3, 3) \cup (3, \infty)$

b) when is $f(x) \geq 0$?
 $[-3, \infty)$