1.5: Parametric Relations and Inverses

Example 1: Defining a Function Parametrically

Consider the set of all ordered pairs (x, y) defined by the equations where t is any real number.

$$x = t + 1$$
$$y = t^2 + 2t$$

a) Find the points determined by $t = -3, -2, -1, 0, 1, 2, 3$.

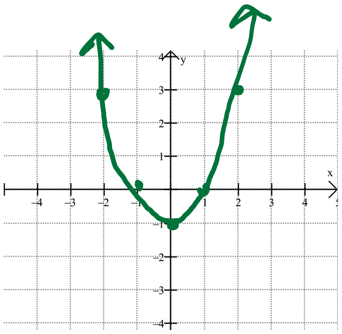
t	$x = t + 1$	$y = t^2 + 2t$	(x, y)
-3	-2	3	$(-2,3)$
-2	-1	0	$(-1,0)$
-1	0	-1	$(0,-1)$
0	1	0	(1.0)
1	2	3	$(2,3)$
2	3	8	$(3,8)$
3	4	15	$(4,15)$

b) Find an algebraic relationship between x and y. (This is often called "eliminating the parameter.") Is y a function of x?

$$x = t + 1$$
$$-1 \quad -1$$
$$t = x - 1$$

$$y = t^2 + 2t$$
$$y = (x-1)^2 + 2(x-1)$$
$$y = (x-1)(x-1) + 2(x-1)$$
$$y = x^2 - x - x + 1 + 2x - 2$$
$$\boxed{y = x^2 - 1}$$

c) Graph the relation in the (x, y) plane.



**Now repeat this problem using a graphing calculator.

Graphing in Parametric Mode on a Calculator

Inverse Relation:

reflect across the line y = x

(x, y) ⟶ (y, x)

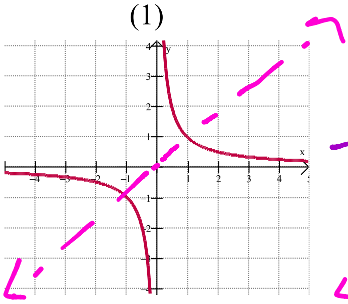
Horizontal Line Test: The inverse of a relation is a function iff (if and only if) each horizontal line intersects the graph and the original in at most one point.

Example 2: Answer the following questions about each graph below. Then graph the inverse of each relation.

a) Is the relation a function?

b) Is the relations inverses a function?

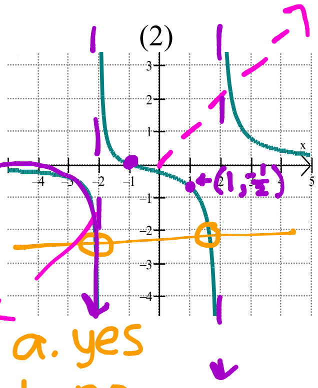
(1)



a. yes

b. yes

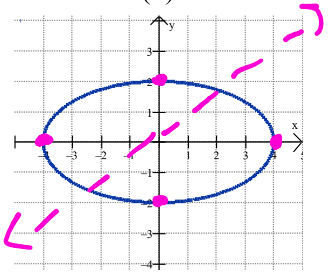
(2)



a. yes

b. no

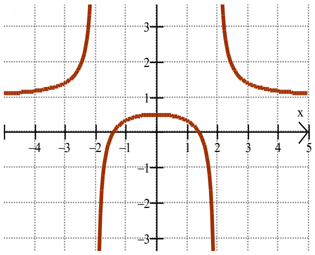
(3)

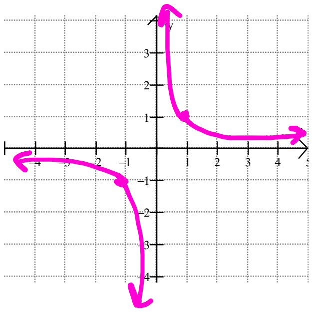


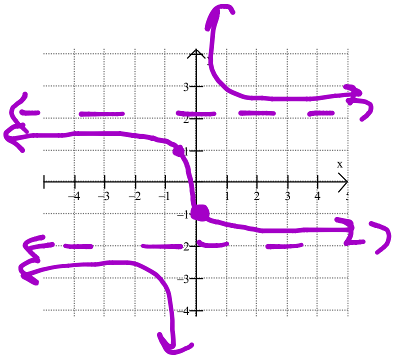
a. no

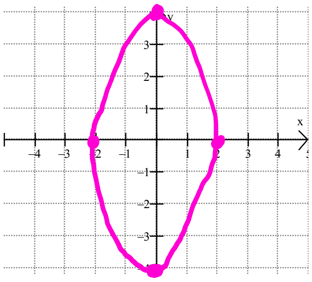
b. no

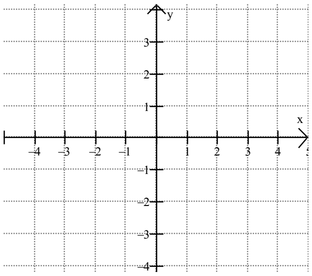
(4)











Finding an inverse algebraically

- Switch "x" and "y"
- Solve for y.

watch out for domain!!

Example 3: Find $f^{-1}(x)$.

a) $\overset{y}{f(x)} = \frac{x}{x+1}$

$(y+1)\,x = \frac{y}{y+1}(y+1)$

$x(y+1) = y$

$xy + x = y$

$\underline{-y \qquad -y}$

$xy - y + x = 0$

$\underline{-x \qquad -x}$

$xy - y = -x$

$\frac{y(x-1)}{x-1} = \frac{-x}{x-1}$

$\boxed{y = \frac{-x}{x-1}}$

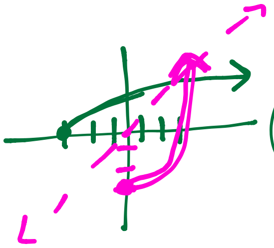
b) $\overset{y}{f(x)} = \sqrt{x+3}$

$(x)^2 = \left(\sqrt{y+3}\right)^2$

$x^2 = y + 3$

$\underline{-3 \qquad -3}$

$\boxed{y = x^2 - 3}$



D: $[-3,\infty)$

R: $[0,\infty)$

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R: $[-3,\infty)$

Inverse Composition Rule

If f and g are inverses,

$(f\circ g) = (g\circ f) = x$

Example 4: Show that $f(x) = \underline{x^3+1}$ and $g(x) = \sqrt[3]{x-1}$ are inverse functions.

$(f\circ g) = f(\underline{g}(x)) = f\left(\sqrt[3]{x-1}\right)$

$= \left(\sqrt[3]{x-1}\right)^3 + 1$

$= x - 1 + 1 = x$ ✓

$(g\circ f) = g(f(x)) = g(x^3+1)$

$= \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$ ✓

f(x) & g(x) are inverses.