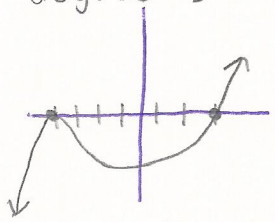
Polynomials Review for Quiz

Sketch a graph of each polynomial without using a calculator.

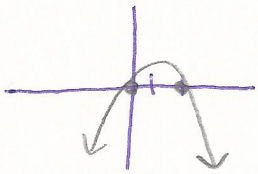
1. $f(x) = (x-3)(x+4)^2$

$x = 3$ mult. 1 (cross)
$x = -4$ mult. 2 (touch)
degree: 3 l.c.: positive



2. $f(x) = -x^3(x-2)$

$x = 0$ mult. 3 (cross)
$x = 2$ mult. 1 (cross)
degree: 4 l.c.: negative



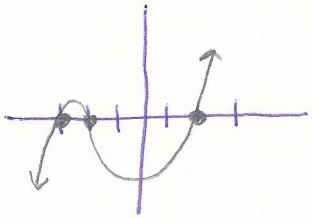
3. $f(x) = x^3 + 3x^2 - 4x - 12$

possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

-2 | 1 | 3 | -4 | -12
↓ | | -2 | -2 | 12
1 | 1 | -6 | 0

$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = -3 \quad x = 2$

zeros: $x = -2$ mult. 1
$x = -3$ mult. 1
$x = 2$ mult. 1
degree: 3 l.c.: positive



Divide using synthetic or long division.

4. $\dfrac{x^3 - 4x^2 + 8x - 8}{x - 2}$

2 | 1 | -4 | 8 | -8
↓ | | 2 | -4 | 8
1 | -2 | 4 | 0

$x^2 - 2x + 4$

5. $\dfrac{4x^4 - 7x^2 + 5x - 3}{2x + 1}$

$$2x+1 \overline{)\,4x^4 + 0x^3 - 7x^2 + 5x - 3}$$

$2x^3 - x^2 - 3x + 4$

$-(4x^4 + 2x^3)$
$-2x^3 - 7x^2$
$-(-2x^3 - x^2)$
$-6x^2 + 5x$
$-(-6x^2 - 3x)$
$8x - 3$
$-(8x + 4)$
-7

$$2x^3 - x^2 - 3x + 4 + \frac{-7}{2x+1}$$

* you have to do long division because of the leading coefficient "2" on $2x+1$.

6. $\frac{x^5+7x^4+x^2-2}{x^2-3}$

$$x^3+7x^2+3x+21+\frac{9x+61}{x^2-3}$$

$$\begin{array}{r} x^3+7x^2+3x+21 \\ x^2+0x-3\overline{)\,x^5+7x^4+0x^3+x^2+0x-2} \\ \underline{x^5+0x^4-3x^3\ \downarrow} \\ 7x^4+3x^3+x^2 \\ \underline{7x^4+0x^3-21x^2} \\ 3x^3+21x^2+0x \\ \underline{3x^3+0x^2-9x} \\ 21x^2+9x-2 \\ \underline{21x^2+0x-63} \\ 9x+61 \end{array}$$

Find all zeros of each polynomial

7. $f(x)=2x^4-7x^3-2x^2-7x-4$

possible: $\pm1, \pm2, \pm4, \pm\frac{1}{2}$

4 | 2 −7 −2 −7 −4
↓ 8 4 8 4
2 1 2 1 | 0

$2x^3+x^2+2x+1=0$

$x^2(2x+1)+1(2x+1)=0$

$(x^2+1)(2x+1)=0$

$x^2+1=0 \quad 2x+1=0$

$x=\pm i \quad x=-\frac{1}{2}$

zeros of $f(x)$: $x=4, x=-\frac{1}{2}, x=i, x=-i$

8. $f(x)=3x^4+10x^3-11x^2-10x+8$

possible: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

1 | 3 10 −11 −10 8
↓ 3 13 2 −8
3 13 2 −8 | 0

−1 | 3 13 2 −8
↓ −3 −10 8
3 10 −8 | 0

$3x^2+10x-8=0$

$(3x-2)(x+4)=0$

$x=\frac{2}{3} \quad x=-4$

zeros of $f(x)$: $x=1, x=-1, x=\frac{2}{3}, x=-4$

9. $f(x)=x^3+2x^2-34x+7$

possible: $\pm1, \pm7$

−7 | 1 2 −34 7
↓ −7 35 −7
1 −5 1 | 0

$x^2-5x+1=0$

$x=\frac{5\pm\sqrt{25-4(1)(1)}}{2}$

$x=\frac{5\pm\sqrt{21}}{2}$

zeros of $f(x)$: $x=-7, x=\frac{5\pm\sqrt{21}}{2}$

10. $f(x)=2x^4+x^3+x^2+x-1$

possible: $\pm1, \pm\frac{1}{2}$

−1 | 2 1 1 1 −1
↓ −2 1 −2 1
2 −1 2 −1 | 0

$2x^3-x^2+2x-1=0$

$x^2(2x-1)+1(2x-1)=0$

$(x^2+1)(2x-1)=0$

$x=\pm i \quad x=\frac{1}{2}$

zeros of $f(x)$: $x=-1, x=\frac{1}{2}, x=i, x=-i$

11. $f(x) = 2x^3 - 9x^2 + 2x + 30$

possible: $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 5, \pm\frac{5}{2}, \pm 6, \pm 10, \pm 15, \pm\frac{15}{2}, \pm 30$

$-\frac{3}{2}$ | 2 | −9 | 2 | 30
↓ | | −3 | 18 | −30
2 | −12 | 20 | 0

$2x^2 - 12x + 20 = 0$

$x = \frac{12 \pm \sqrt{144 - 4(20)(2)}}{4}$

$x = \frac{12 \pm 4i}{4} = 3 \pm i$

zeros of $f(x)$: $x = -\frac{3}{2}, x = 3+i, x = 3-i$

12. $f(x) = 6x^4 + 11x^3 - 16x^2 - 11x + 10$

possible: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}, \pm 5, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm 10, \pm\frac{10}{3}$

1 | 6 | 11 | −16 | −11 | 10
↓ | | 6 | 17 | 1 | −10
6 | 17 | 1 | −10 | 0

−1 | 6 | 17 | 1 | −10
↓ | | −6 | −11 | 10
6 | 11 | −10 | 0

$6x^2 + 11x - 10 = 0$

$(3x-2)(2x+5) = 0$

$3x - 2 = 0$ $\quad$ $2x + 5 = 0$

$x = \frac{2}{3}$ $\quad$ $x = -\frac{5}{2}$

zeros of $f(x)$: $x = 1, x = -1, x = \frac{2}{3}, x = -\frac{5}{2}$

write a polynomial in standard form whose zeros and their multiplicities include those listed.

13. Degree: 3; zeros: $\sqrt{5}, -\sqrt{5}, 3$

$(x-\sqrt{5})(x+\sqrt{5})(x-3) = (x^2-5)(x-3) = x^3 - 3x^2 - 5x + 15$

14. Degree: 3; zeros: $1+i, 2$

$(x-1-i)(x-1+i)(x-2) = (x^2-2x+2)(x-2) = x^3 - 2x^2 - 2x^2 + 4x + 2x - 4$

$= x^3 - 4x^2 + 6x - 4$

15. Degree: 4; zeros: -2 mult. 2; 4 mult 2

$(x+2)(x+2)(x-4)(x-4) = (x^2+4x+4)(x^2-8x+16)$

$= x^4 - 8x^3 + 16x^2 + 4x^3 - 32x^2 + 64x + 4x^2 - 32x + 64$

$= x^4 - 4x^3 - 12x^2 + 32x + 64$