

Precalculus: Polynomial, Power, and Rational Functions

2.1: Linear and Quadratic Function Modeling

Polynomial Functions: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

n is a nonnegative integer, $a_n \neq 0$

A polynomial function is defined and continuous for all real numbers.

Name	Form	Degree
Zero Function	$f(x) = 0$	undefined
Constant Function	$f(x) = c$	degree = 0
Linear Function	$f(x) = mx + b$	degree = 1
Quadratic Function	$f(x) = ax^2 + bx + c$	degree = 2

Examples: Determine which of the following are polynomial functions. For those that are not explain why.

a) $f(x) = 4x^3 - 5x - \frac{1}{2}$ **yes**
degree: 3 **L.C.: 4**

b) $g(x) = (x^{-4}) + 7 = \frac{6}{x^4} + 7$
NOT a polynomial function

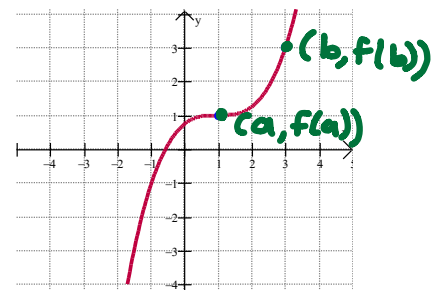
c) $h(x) = \sqrt{9x^4 + 16x^2}$
Not a polynomial function.

d) $k(x) = 15x - 2x^4$ **yes**
degree: 4
L.C.: -2

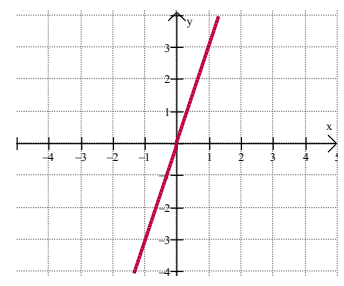
Average Rate of Change:

Slope

$$\frac{f(b) - f(a)}{b - a}$$

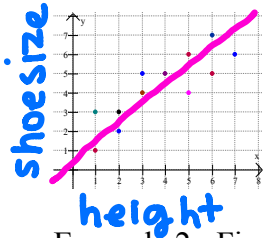


Constant Rate of Change:

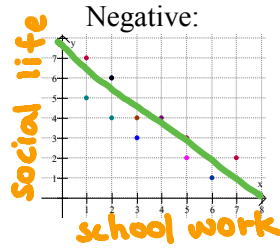


Linear Correlation/Correlation Coefficient:

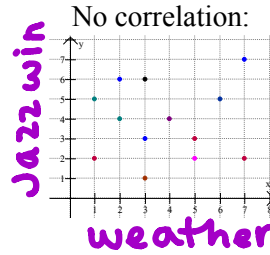
Positive:



Negative:



No correlation:



Example 2: Find the max/min (vertex) and the zeros (x-intercepts) of the function by hand. Then graph on your calculator to check.

a) $y = x^2 - 7x + 10$

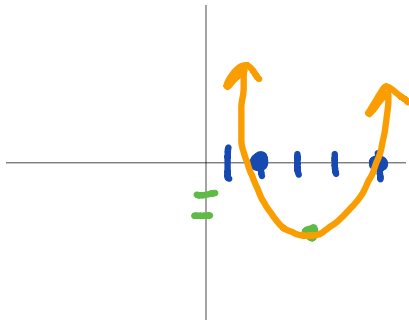
$y = (x-2)(x-5)$

zeros: $(2, 0)$ & $(5, 0)$

$\frac{2+5}{2} = \frac{7}{2} = 3.5$

$y = (3.5)^2 - 7(3.5) + 10$

$y = -2.25$ vertex: $(3.5, -2.25)$



b) $y = -x^2 - 4x + 3$

$y = (-x^2 - 4x + \underline{\quad}) + 3 - \underline{\quad}$

$y = -(x^2 + 4x + \frac{4}{4}) + 3 - (-1) \frac{4}{4}$

$-\frac{1}{2} \cdot 2 \quad (2)^2 = 4$
 $-(x^2 + 4x + 4) + 3 + (-4)$
 $= -(x+2)^2 + 7$ vertex: $(-2, 7)$

$0 = -(x+2)^2 + 7$

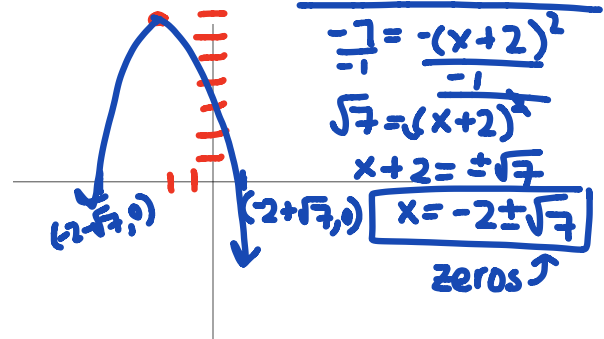
$-7 = -(x+2)^2$

$\sqrt{7} = (x+2)$

$x+2 = \pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

zeros ↗



c) $y = x^2 + 6x + 11$

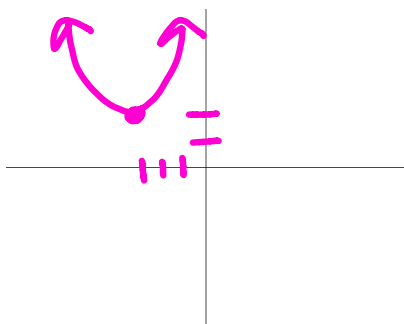
$y = (x^2 + 6x + \underline{\quad}) + 11 - \underline{\quad}$

$y = (x^2 + 6x + 9) + 11 - 9$

$y = (x+3)^2 + 2$

vertex: $(-3, 2)$

no x-intercepts



d) $y = 2x^2 + 6x - 3$

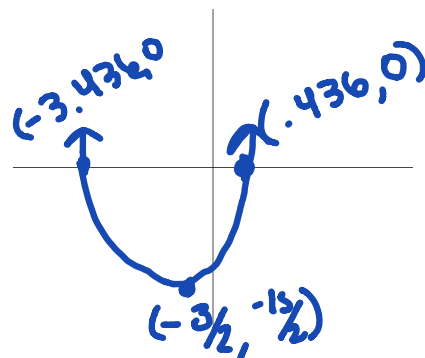
$y = (2x^2 + 6x + \underline{\quad}) - 3 - \underline{\quad}$

$y = 2(x^2 + 3x + \frac{9}{4}) - 3 - (2) \frac{9}{4}$

$y = 2(x + \frac{3}{2})^2 - 3 - \frac{9}{2} \quad 2(\frac{9}{4}) = \frac{18}{4}$

$y = 2(x + \frac{3}{2})^2 - \frac{15}{2}$

vertex: $(-\frac{3}{2}, -\frac{15}{2})$



Writing Equations of Quadratic Functions

Case 1 When given the vertex and a point on the parabola.

Steps: Use vertex form: $y = a(x-h)^2 + k$

1. Plug in the vertex h and k
2. Plug in the point for x and y .
3. Solve for a .
4. Write the equation in vertex form using the vertex and a .

Example 3: Write the equation of the quadratic.

- a) vertex (h, k) $(3, 4)$ and goes through the point (x, y) $(0, 1)$

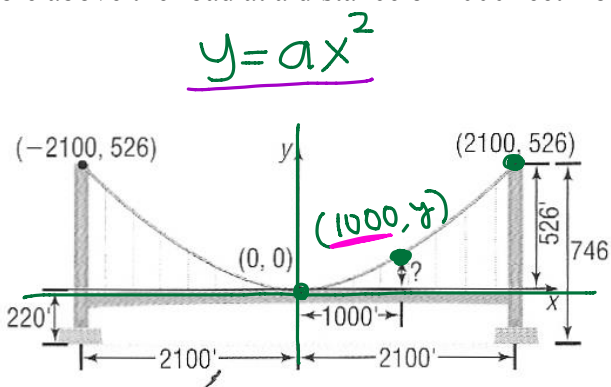
$$y = a(x-h)^2 + k$$

$$1 = a(0-3)^2 + 4$$

$$\begin{aligned} -4 &= 9a + 4 \\ -8 &= 9a \\ a &= -\frac{8}{9} \end{aligned}$$

$$y = -\frac{8}{9}(x-3)^2 + 4$$

- b) Application: The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its 746-foot-tall towers are 4200 feet apart. The bridge is suspended from two huge cables more than 3 feet in diameter; the 90-foot-wide roadway is 220 feet above the water. The cables are parabolic in shape and touch the road surface at the center of the bridge. Find the height of the cable above the road at a distance of 1000 feet from the center.



$$y = ax^2$$

$$526 = a(2100)^2$$

$$a = 1.19 \times 10^{-4}$$

$$y = (1.19 \times 10^{-4})(1000)^2$$

$$y \approx 119.27$$

Example 4: Write an equation for the linear function f satisfying the given conditions. Graph $y = f(x)$.

- a) $f(-5) = -1$ and $f(2) = 4$

$$y = mx + b$$

$$\begin{aligned} x &= -5 \\ y &= -1 \end{aligned}$$

$$(-5, -1)$$

$$(2, 4)$$

$$m = \frac{4 - (-1)}{2 - (-5)} = \frac{5}{7}$$

$$y = mx + b$$

$$4 = \frac{5}{7}(2) + b$$

$$4 = \frac{10}{7} + b$$

$$4 - \frac{10}{7} = b$$

$$\frac{28}{7} - \frac{10}{7} = b$$

$$\frac{18}{7} = b$$

$$y = \frac{5}{7}x + \frac{18}{7}$$