2.3: Polynomial Functions of Higher Degree with Modeling

POLYNOMIALS:

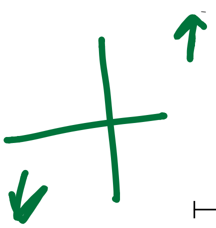
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_o$$

- A polynomial written in this way, with terms in descending degree, is written in **standard form**.
- The constants $a_n, a_{n-1}, \ldots, a_0$ are the **coefficients** of the polynomial.
- The term $a_n x^n$ is the **leading coefficient**, a_0 is the **constant term**.

Investigating End Behavior of Polynomials

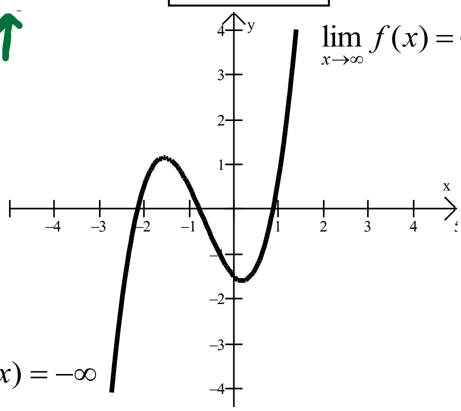
For any polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_o$, the limits $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$ are determined by the **degree** n of the polynomial and its **leading coefficient** a_n.

degree is odd
l.c. is positive



$a_n > 0$
n odd

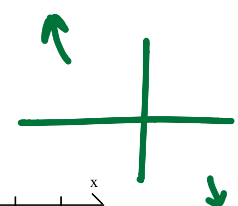
$\lim\limits_{x\to\infty} f(x) = \infty$



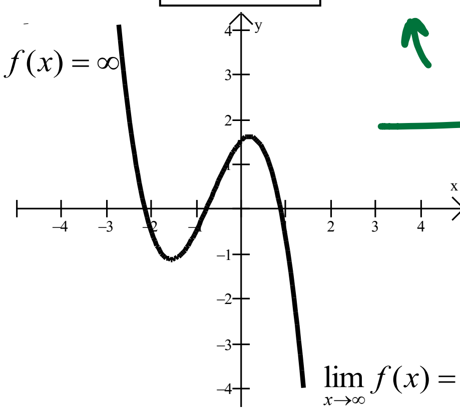
$\lim\limits_{x\to-\infty} f(x) = -\infty$

$a_n < 0$
n odd

degree is odd
l.c. is negative

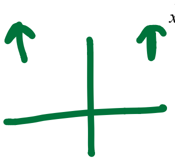


$\lim\limits_{x\to-\infty} f(x) = \infty$



$\lim\limits_{x\to\infty} f(x) = -\infty$

degree is even
l.c. is positive



$a_n > 0$
n even

$\lim\limits_{x\to-\infty} f(x) = \infty$

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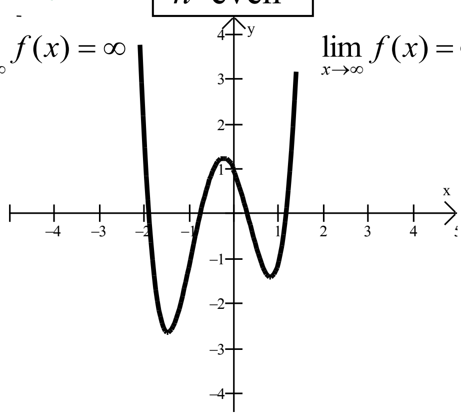
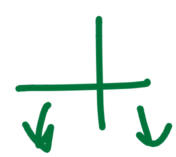
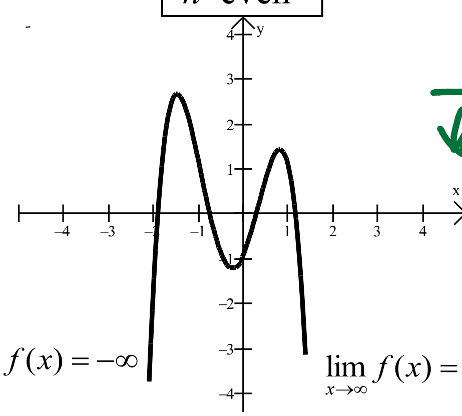


$a_n < 0$
n even

degree is even
l.c. is negative





$\lim\limits_{x\to-\infty} f(x) = -\infty$

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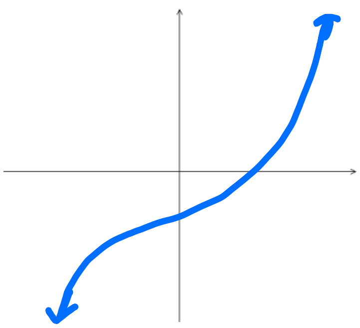
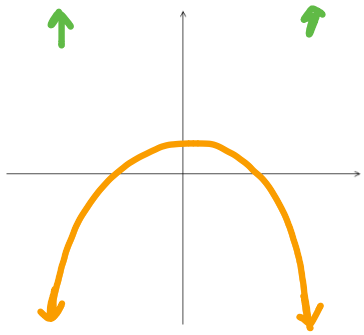
Example 1: Sketch a graph for each polynomial without using a calculator. Don't worry about the number of turns that the polynomial has just worry about the end behavior.

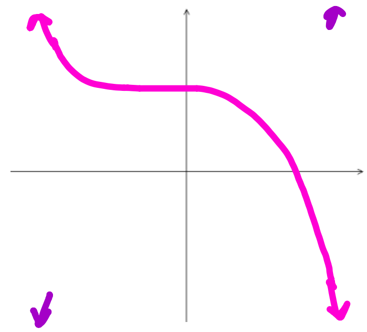
a) $f(x) = x^3 + 6x - 9$



b) $f(x) = -2x^4 + x^2 - 9x + 1$



c) $f(x) = -3x^5 + 2x^2 + 6$



Zeros of a Polynomial
Recall that zeros refer to the x-intercepts of a graph.

To find the zeros of a polynomial

1. Set the polynomial equal to zero.
2. Solve for x.

Example 2: Find the zeros of the polynomial function. Then sketch the graph.

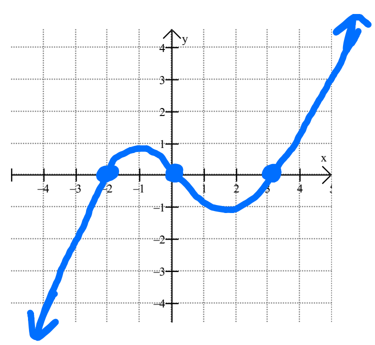
a) $f(x) = x^3 - x^2 - 6x$

$0 = x^3 - x^2 - 6x$

$0 = x(x^2 - x - 6)$

$0 = x(x-3)(x+2)$

$x = 0 \quad x = 3 \quad x = -2$



Multiplicity:
If a factor (x – r) occurs more than once, r is called a repeated or multiple zero of *f*. The multiplicity of *f* is number of times the zero repeats.

Even multiplicity: The graph **touches** the x-axis
Odd multiplicity: The graph **crosses** the x-axis

$(x-2)(x-2) = (x-2)^2$

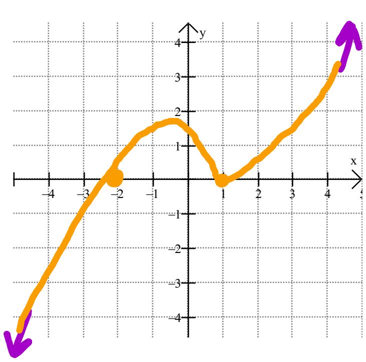
$x = 2$

Example 3: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of the polynomial function.

a) $f(x) = (x+2)^3(x-1)^2$

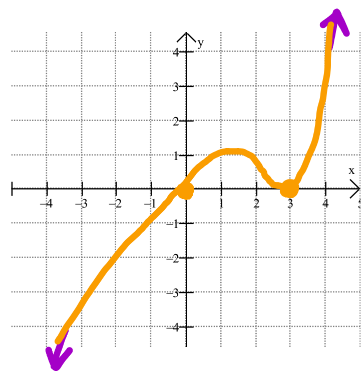
$x=-2$ mult. 3
cross
$x=1$ mult. 2
touch

degree: 5



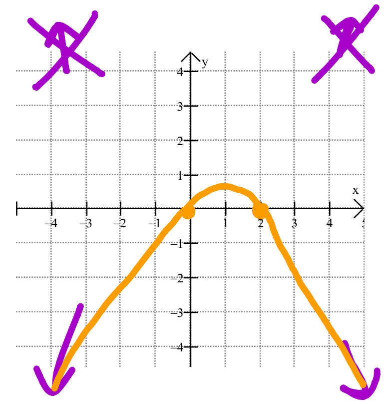
b) $f(x) = x(x-3)^2$

degree: 3
$x=0$ mult. 1
cross
$x=3$ mult. 2
touch



c) $f(x) = -x^3(x-2)$

degree: 4
$x=0$ mult. 3
cross
$x=2$ mult. 1
cross



Example 4: Write a polynomial in standard form with the given zeros.

a) zeros: -3, 0, 2

$f(x) = (x+3)(x-0)(x-2)$

$= x(x+3)(x-2)$

$= x(x^2+x-6)$

$\boxed{f(x) = x^3+x^2-6x}$