

## 2.7: Solving Rational Equations in One Variable

### Least Common Denominators

Example 1: Find the least common denominator.

a)  $\frac{3}{(x+2)(x-2)}, \frac{x-1}{x+2}$

LCD:  $(x+2)(x-2)$

b)  $\frac{x^2+10}{2x^2+2x}, \frac{6}{x^2+7x+6}$   
 $\frac{x^2+10}{2x(x+1)}, \frac{6}{(x+1)(x+6)}$

LCD:  $(x+1)(2x)(x+6)$

#### Steps for Solving a Rational Equation

1. We begin by finding the LCD of all the terms of the equation.
2. We clear the denominators from the equation by multiplying each side of the equation by the LCD.
3. We must check each solution of the resulting equation in the original equation.  
 If a solution to the resulting equation is not a solution to the original equation then it is called an extraneous solution.

Example 2: Solve each rational equation.

a)  $\left(x + \frac{3}{x} = 4\right) \times$

LCD:  $x$

$$\begin{aligned} x^2 + 3 &= 4x \\ -4x &\quad -4x \\ \hline x^2 - 4x + 3 &= 0 \\ (x-3)(x-1) &= 0 \\ x=1 \quad x=3 & \end{aligned}$$

Check:

$x=1$	$x=3$
$1 + \frac{3}{1} = 4$	$3 + \frac{3}{3} = 4$
$1 + 3 = 4$	$3 + 1 = 4$
$4 = 4$	$4 = 4$
✓	✓

$x=1$  and  $x=3$  are solutions to the original equation.

b)  $\left(x + \frac{1}{x-4} = 0\right) \times (x-4)$

LCD:  $(x-4)$

$$\begin{aligned} x(x-4) + 1 &= 0 \\ x^2 - 4x + 1 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} \\ &= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} \end{aligned}$$

$x = 2 \pm \sqrt{3}$

Check your zeros by using a calculator

$x = 2 \pm \sqrt{3}$  are solutions to the original equation.

c)  $\left(\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}\right) (x-1)(x-3)$

LCD:  $(x-1)(x-3)$

$2x(x-3) + 1(x-1) = 2$

$2x^2 - 6x + x - 1 - 2 = 0$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$x = -\frac{1}{2} \quad x = 3$

Check:

$x = -\frac{1}{2}$

$-\frac{2(-\frac{1}{2})}{-\frac{1}{2}-1} + \frac{1}{-\frac{1}{2}-3} = \frac{2}{(-\frac{1}{2})^2 - 4(-\frac{1}{2}) + 3}$

$\frac{-1}{-\frac{3}{2}} + \frac{1}{-\frac{7}{2}} = \frac{2}{\frac{1}{4} + 2 + 3} = \frac{2}{\frac{21}{4}}$

$\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$

$\frac{14}{21} - \frac{6}{21} = \frac{8}{21}$

$\frac{8}{21} = \frac{8}{21} \checkmark$

$x = 3$

$\frac{2(3)}{3-1} - \frac{1}{3-3} = \frac{2}{(3)^2 - 4(3) + 3}$

undefined

$x = -\frac{1}{2}$  is a solution to the original equation  
 $x = 3$  is an extraneous solution.

d)  $\left(\frac{x-3}{x} + \frac{3}{x+2} + \frac{6}{x^2+2x} = 0\right) x(x+2)$

LCD:  $x(x+2)$

$(x-3)(x+2) + 3x + 6 = 0$

$x^2 - x - 6 + 3x + 6 = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$

Check:

$x = 0$

$\frac{0-3}{0} + \frac{3}{0+2} + \frac{6}{0+0} = 0$

undefined

$x = -2$

$\frac{-2-3}{-2} + \frac{3}{-2+2} + \frac{6}{(-2)^2 + 2(-2)} = 0$

undefined

$x = 0$  and  $x = -2$  are extraneous solutions

### Applications:

#### Finding a Minimum Perimeter:

Find the dimensions of the rectangle with minimum perimeter if its area is 200 square meters. Find this least perimeter.