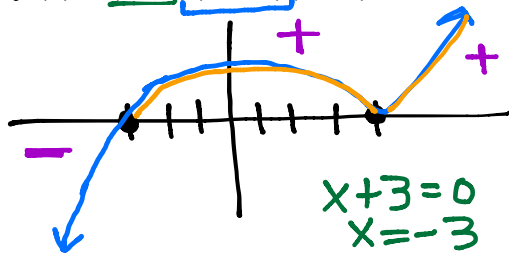


2.8: Solving Inequalities in One Variable

Example 1: Graph the polynomial to determine where it is Zero, Positive, or Negative

a) $f(x) = (x+3)(x^2+1)(x-4)^2$



zero: $x = -3, x = 4$
 positive: $(-3, 4) \cup (4, \infty)$
 negative: $(-\infty, -3)$

$x+3=0$
 $x=-3$

$x^2+1=0$
 $x=\pm i$

$x-4=0$
 $x=4$
 mult. 2

Example 2: Solve inequalities

Use what you found in the previous problem to solve the inequality.

a) $(x+3)(x^2+1)(x-4)^2 \geq 0$

$(-3, 4) \cup (4, \infty)$

b) $(x+3)(x^2+1)(x-4)^2 \geq 0$

$[-3, \infty)$

c) $(x+3)(x^2+1)(x-4)^2 < 0$

$(-\infty, -3)$

d) $(x+3)(x^2+1)(x-4)^2 \leq 0$

$(-\infty, -3]$

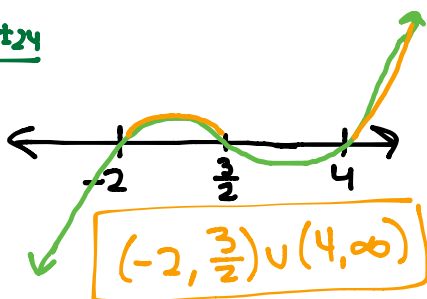
Example 3: Solving a Polynomial Inequality Analytically

a) $2x^3 - 7x^2 - 10x + 24 > 0$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$
 q : $\pm 1, \pm 2$

$4 \overline{) 2 \ -7 \ -10 \ 24}$
 $\downarrow \ 8 \ 4 \ -24$
 $2 \ 1 \ -6 \ 0$

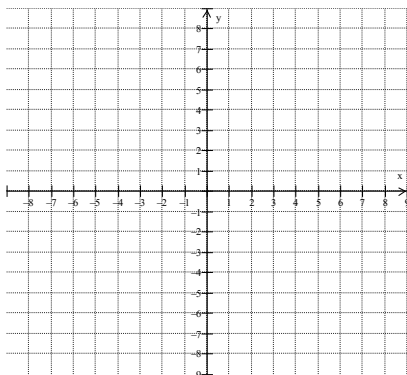
$2x^2 + x - 6 = 0$
 $(2x-3)(x+2) = 0$
 $x = 3/2 \quad x = -2$



$(-2, 3/2) \cup (4, \infty)$

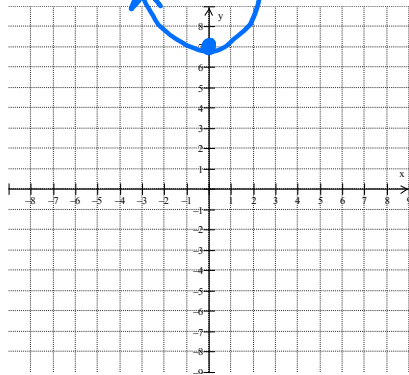
Example 4: Solve a Polynomial Inequality Graphically

a) $x^3 - 6x^2 \leq 2 - 8x$



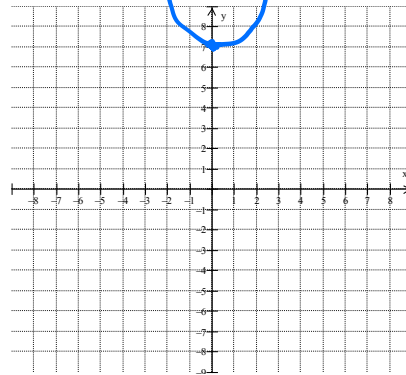
$$f(x) = (x^2 + 7)(2x^2 + 1) > 0$$

$$(0+7)(2+1) = (7)(1) = 7$$



$(-\infty, \infty)$

$$c) (x^2 + 7)(2x^2 + 1) < 0$$



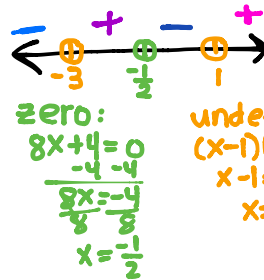
never

Example 5: Solving a Rational Inequality

$$a) \frac{(x-1)5}{(x-1)(x+3)} + \frac{3(x+3)}{(x-1)(x+3)} < 0 \quad \text{LCD: } (x+3)(x-1)$$

$$\frac{5(x-1) + 3(x+3)}{(x-1)(x+3)}$$

$$\frac{5x-5+3x+9}{(x-1)(x+3)} = \frac{8x+4}{(x-1)(x+3)} < 0$$



zero:
 $8x+4=0$
 $-4-4$
 $8x=-4$
 $x=-\frac{1}{2}$

undefined:
 $(x-1)(x+3)=0$
 $x-1=0 \quad x+3=0$
 $x=1 \quad x=-3$

$$x = -4 \quad \frac{-}{(-)(-)} = \frac{-}{+} = -$$

$$x = -2 \quad \frac{-}{(-)(+)} = \frac{-}{-} = +$$

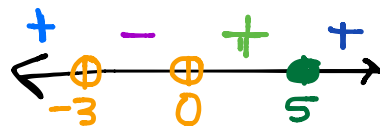
$$x = 0 \quad \frac{+}{(-)(+)} = \frac{+}{-} = -$$

$$x = 2 \quad \frac{+}{(+)(+)} = \frac{+}{+} = +$$

$$\boxed{(-\infty, -3) \cup (-\frac{1}{2}, 1)}$$

$$b) \frac{(x-5)^4}{x(x+3)} \geq 0$$

$$\frac{(x-5)^4}{x(x+3)} \geq 0$$



zeros:
 $(x-5)^4 = 0$
 $x-5=0$
 $x=5$

undefined:
 $x(x+3)=0$
 $x=0 \quad x=-3$

$$x = -4 \quad \frac{+}{(-)(-)} = \frac{+}{+} = +$$

$$x = 6 \quad \frac{+}{(+)(+)} = \frac{+}{+} = +$$

$$x = 1 \quad \frac{+}{(-)(+)} = \frac{+}{-} = -$$

$$x = 1 \quad \frac{+}{(+)(+)} = \frac{+}{+} = +$$

$$\boxed{(-\infty, -3) \cup (0, \infty)}$$