**PRECALCULUS**
**Chapter 2 Test REVIEW**

**Name:** Jaussi
**Period** ______ **Desk** ______

**SHOW ALL YOUR WORK on a separate sheet of paper.** **Score** ______

1. Write each quadratic in vertex form. Identify the vertex. Then solve to find the $x$-intercepts.

a) $y = x^2 - 18x + 100$ COMPLETE THE SQUARE.

$y = (x^2 - 18x + \_\_)+100 - \_\_$

$\frac{-18}{2} = -9 \quad (-9)^2 = 81$

$y = (x^2 - 18x + 81) + 100 - 81$

$\boxed{y = (x-9)^2 + 19}$

$\boxed{\text{vertex}: (9, 19)}$

$\boxed{\text{x-intercepts: none}}$

$0 = (x-9)^2 + 19$

$-19 = (x-9)^2 \rightarrow$ this will result in imaginary zeros.

b) $y = -2x^2 - 16x - 20$

$y = -2(x^2 + 8x + \underline{16}) - 20 - (-2)\,\underline{16}$

$\frac{8}{2} = 4 \quad (4)^2 = 16$

$y = -2(x+4)^2 - 20 + 32$

$\boxed{y = -2(x+4)^2 + 12}$

$\boxed{\text{vertex}: (-4, 12)}$

$0 = -2(x+4)^2 + 12$

$-12 = -2(x+4)^2$

$6 = (x+4)^2$

$x + 4 = \pm\sqrt{6}$

$x = -4 \pm 6$

$\boxed{\text{x-intercepts}: (-4+\sqrt{6}, 0)\ (-4-\sqrt{6}, 0)}$

2. Write an equation for the quadratic function whose graph contains the given vertex and point. (HINT: use the formula for a quadratic in vertex form.) $y = a(x-h)^2 + k$

a) Vertex: (2, -7), point (0, 5)
h k x y

$5 = a(0-2)^2 - 7$

$12 = a(-2)^2$

$12 = a(4)$

$a = 3$

$\boxed{y = 3(x-2)^2 - 7}$

b) Vertex: (-2, -5), point (-4, -27)
h k x y

$-27 = a(-4+2)^2 - 5$

$-22 = a(-2)^2$

$-22 = 4a$

$a = \frac{-11}{2}$

$\boxed{y = \frac{-11}{2}(x+2)^2 - 5}$

3. Use limit notation to describe the end behavior of the polynomial function.

a) $f(x) = -3x^4 - 6x - 300$

degree: 4
leading coefficient: −

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

b) $f(x) = 5x^3 + 2x^2 + .01x + 25$

degree: 3
leading coefficient: +

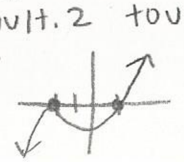
$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

4. State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses or touches the $x$-axis at the corresponding $x$-intercept. Then sketch the graph by hand.

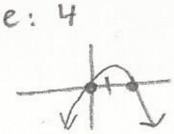
a) $f(x) = (x-1)^3(x+2)^2$

x=1 mult. 3 crosses
x=-2 mult. 2 touches
degree: 5



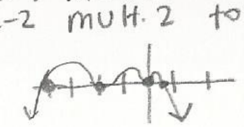
b) $f(x) = -x^3(x-2)$

x=0 mult. 3 cross
x=2 mult. 1 cross
degree: 4

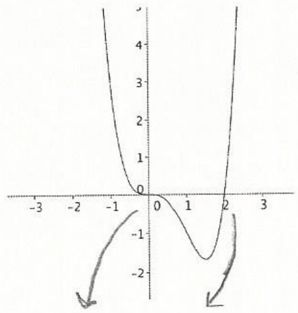


c) $f(x) = -3x^2(2x-1)(x+4)(x+2)^2$

x=0 mult. 2 touch
x=1/2 mult. 1 cross
x=-4 mult. 1 cross
x=-2 mult. 2 touch
degree: 6



5. Match the graph to the equation.




A. $x^2(x-2)^2$

B. $-x(x-2)^3$

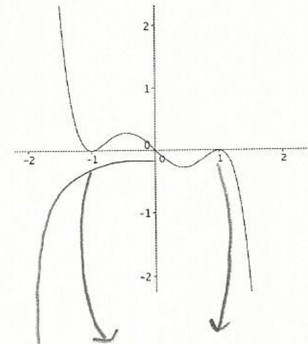
C. $x(x+2)^3$

D. $x^2(x+2)^2$

E. $x^3(x-2)$

zero at x = 0 and x = 2 crosses the graph so that has an odd multiplicities.

only equation with odd multiplicity with both x = 0 and x = 2.




A. $-x(x-1)^2(x+1)^2$

B. $x(x-1)(x+1)$

C. $x^3(x-1)(x+1)$

D. $-x^2(x-1)(x+1)$

E. $-x^2(x-1)^2(x+1)$

zero at x = -1 and x = 1 touches the graph so that has an even multiplicity.

zero at x = 0 crosses the graph so that has an odd multiplicity.

6. Find **all** the zeros (real and imaginary) of the function. (HINT: start with the possible zeros)

a. $f(x)=x^5+8x^4+16x$

Skip

b. $f(x)=2x^3-3x^2-4x+6$

possible: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 3/2 & 2 & -3 & -4 & 6 \\ & \downarrow & 3 & 0 & -6 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

$2x^2-4=0$

$2x^2=4$

$x^2=2$

$x=\pm\sqrt{2}$

$\boxed{\text{zeros: } x=\frac{3}{2}, \pm\sqrt{2}}$

c. $f(x)=x^3-6x^2+7x+4$

possible: $\pm1, \pm2, \pm4$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & \downarrow & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2-2x-1=0$

$x=\frac{2\pm\sqrt{4-4(1)(-1)}}{2(1)}$

$=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm2\sqrt{2}}{2}$

$x=\frac{1\pm\sqrt{2}}{2}$

$\boxed{\text{zeros: } x=4, 1\pm\sqrt{2}}$

d. $f(x)=x^3-x^2+49x-49$

$x^2(x-1)+49(x-1)$

$(x-1)(x^2+49)=0$

$x-1=0$

$x=1$

$x^2+49=0$

$x^2=-49$

$x=\pm7i$

$\boxed{\text{zeros: } x=1, \pm7i}$

e. $f(x)=x^3-x^2-x-2$

possible: $\pm1, \pm2$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & \downarrow & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$x^2+x+1=0$

$x=\frac{-1\pm\sqrt{1-4(1)(1)}}{2}$

$x=\frac{-1\pm\sqrt{-3}}{2}$

$x=\frac{-1\pm i\sqrt{3}}{2}$

$\boxed{\text{zeros: } x=2, \frac{-1\pm i\sqrt{3}}{2}}$

7. Rewrite the polynomial in **factored form** (ie. as the product of linear and irreducible quadratics with real coefficients.)

a) $f(x) = 2x^3 - x^2 + 2x - 3$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 2 & -3 \\ & \downarrow & 2 & 1 & 3 \\ \hline & 2 & 1 & 3 & 0 \end{array}$$

x=1 is a zero means x-1 is a factor.

$2x^2 + x + 3$

↓

since this quadratic cannot be factored, it is irreducible.

$\boxed{f(x) = (x-1)(2x^2+x+3)}$

b) $f(x) = x^3 - 7x^2 + 17x - 15$

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 17 & -15 \\ & \downarrow & 3 & -12 & 15 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

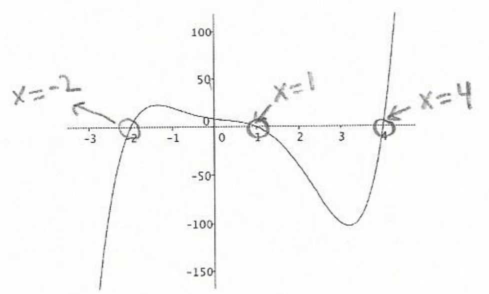
x=3 is a zero means x-3 is a factor.

$x^2 - 4x + 5$ → this quadratic cannot be factored, it is irreducible.

$\boxed{f(x) = (x-3)(x^2-4x+5)}$

8. Use the graph to find the zeros of *f(x)*. Verify all rational zeros using synthetic division. Then rewrite the polynomial in factored form.

a)




$f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$

↓

since this polynomial has a degree 5, we know there are 5 total zeros.

$$\begin{array}{r|rrrrrr} -2 & 1 & -3 & -5 & 5 & -6 & 8 \\ & \downarrow & -2 & 10 & -10 & 10 & -8 \\ \hline 1 & 1 & -5 & 5 & -5 & 4 & 0 \\ & \downarrow & 1 & -4 & 1 & -4 & \\ \hline 4 & 1 & -4 & 1 & -4 & 0 & \\ & \downarrow & 4 & 0 & 4 & & \\ \hline & 1 & 0 & 1 & 0 & & \end{array}$$

x=-2 is a zero.

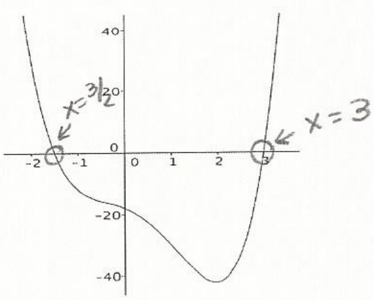
x=1 is a zero

x=4 is a zero

$x^2 + 1 = 0$

$\boxed{f(x) = (x+2)(x-1)(x-4)(x^2+1)}$

b)




$f(x) = 2x^4 - 3x^3 - 5x^2 - 6x - 18$

$$\begin{array}{r|rrrrr} 3 & 2 & -3 & -5 & -6 & -18 \\ & \downarrow & 6 & 9 & 12 & 18 \\ \hline -3/2 & 2 & 3 & 4 & 6 & 0 \\ & \downarrow & -3 & 0 & -6 & \\ \hline & 2 & 0 & 4 & 0 & \end{array}$$

$2x^2 + 4 = 0$

$\frac{2(x^2+2)}{2} = \frac{0}{2}$

$\boxed{f(x) = (x-3)(2x+3)(x^2+2)}$

9. Write a polynomial with the following zeros. You may leave your answer in factored form.

a) degree: 3, zeros: 1 (multiplicity 2) and -2 (multiplicity 1)

$f(x) = (x-1)^2(x+2)$ ← This is factored form. If you want to put in standard form, FOIL.

$f(x) = (x^2-2x+1)(x+2)$

$= x^3+2x^2-2x^2-4x+x+2$

$\boxed{f(x) = x^3 - 3x + 2}$ → Standard FORM

b) degree: 4, zeros: 1, 2, $1 \pm i$

$f(x) = (x-1)(x-2)(x-1+i)(x-1-i)$

$f(x) = (x-1)(x-2)(x^2-2x+1+1)$

$\boxed{f(x) = (x-1)(x-2)(x^2-2x+2)}$ → Factored FORM

c) degree: 3, zeros: -2, $1 \pm i\sqrt{2}$

$f(x) = (x+2)(x-1+i\sqrt{2})(x-1-i\sqrt{2})$

$(x+2)(x^2-2x+1+2)$

$\boxed{f(x) = (x+2)(x^2-2x+3)}$

10. One root of $x^3 - 11x^2 + 29x - 7$ is $2+\sqrt{3}$. Find the other two roots.

$2-\sqrt{3}$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = x^2-4x+4-3$

$= x^2-4x+1$

$$\begin{array}{r} x - 7 \\ x^2-4x+1 \overline{\smash{)}\, x^3 - 11x^2 + 29x - 7} \\ \underline{-x^3 + 4x^2 + -x} \\ -7x^2 + 28x - 7 \\ \underline{-7x^2 + 28x - 7} \\ 0 \end{array}$$

$x - 7 = 0$

$x = 7$

$\boxed{x = 7, \ x = 2 \pm \sqrt{3}}$

11. Consider $f(x) = x^4 + 3x^3 - 11x^2 - 3x + 10$

a) Name all possible combinations of the number of real and imaginary zeros.

4 real zeros
2 real zeros, 2 imaginary zeros
4 imaginary zeros.

b) List all possible rational zeros.

$\pm 1, \pm 2, \pm 5, \pm 10$

c) Find all zeros of $f(x)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 3 | -11 | -3 | 10 |
| | ↓ | 1 | 4 | -7 | -10 |
| 2 | 1 | 4 | -7 | -10 | 0 |
| | | 2 | 12 | 10 | |
| | 1 | 6 | 5 | 0 | |

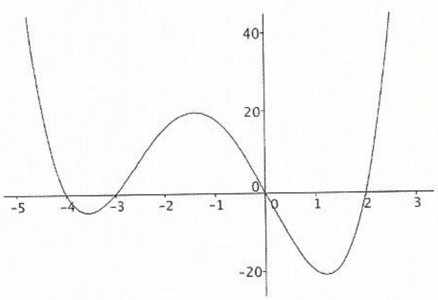
$\boxed{\text{zeros: } x = 1,\ x = 2,\ x = -5,\ x = -1}$

$x^2 + 6x + 5 = 0$
$(x+5)(x+1) = 0$
$x = -5 \quad x = -1$

12. The graph of a polynomial intersects the *x*-axis at two distinct points. Does this mean that the polynomial is a quadratic?

Not necessarily. The polynomial could be a higher degree polynomial with repeated or imaginary roots.

13. Let *f(x)* be the polynomial that is graphed below.



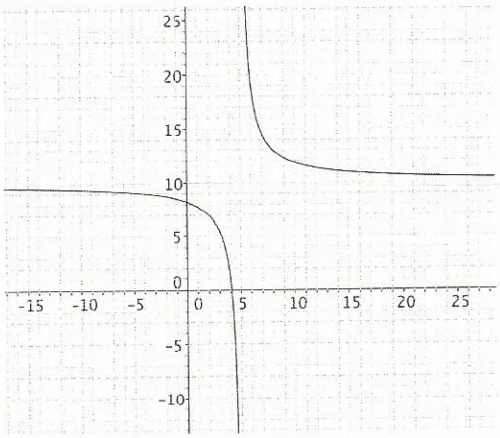

a) Explain how we know that -4 is not a double root.
The graph crosses the graph at $x = -4$ so the multiplicity of the zero is odd.

b) What is the minimum possible degree of *f*? Explain your answer.
The minimum possible degree of f is 4 since there are 4 zeros.

c) Could it be a 5th degree polynomial? What about a sixth degree polynomial? Explain your answer.
No, it cannot be a 5th degree polynomial because the degree of f(x) must be even. we know this from the end behavior. It could be a sixth degree polynomial because the degree six is even.

State the domain and range of each function. Identify any asymptotes or holes.

14.




Domain: $(-\infty,5)\cup(5,\infty)$

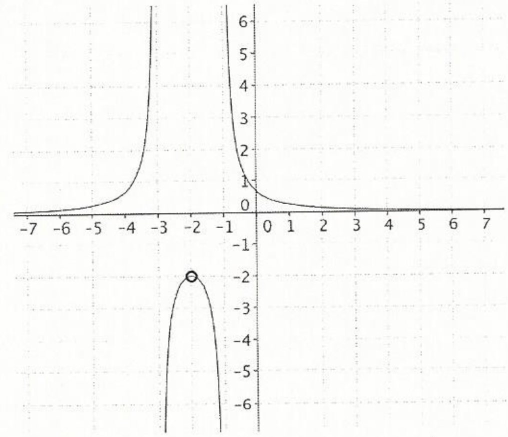
Range: $(-\infty,10)\cup(10,\infty)$

Vertical Asymptote(s): $x=5$

Horizontal Asymptote: $y=10$

Hole(s): none

15.




Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

Range: $(-\infty,-2)\cup(0,\infty)$

Vertical Asymptote(s): $x=-3$ $x=-1$

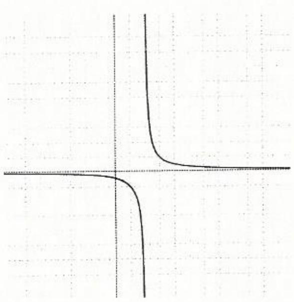
Horizontal Asymptote: $y=0$

Hole(s): $(-2,-2)$

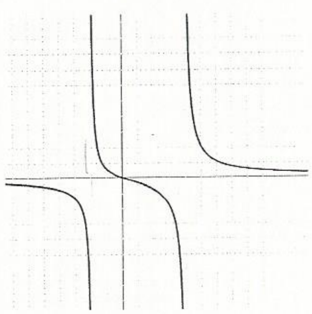
16. Match the graph to the equation.

A) $f(x)=\dfrac{1}{(x-2)}$ B) $f(x)=\dfrac{x-2}{(x+3)(x-1)}$ C) $f(x)=\dfrac{x^2+2}{x^2}$ D) $f(x)=\dfrac{3x}{x^2-2x-8}$

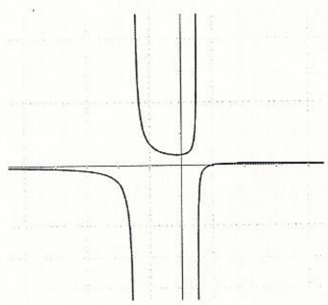
$(x-4)(x+2)$



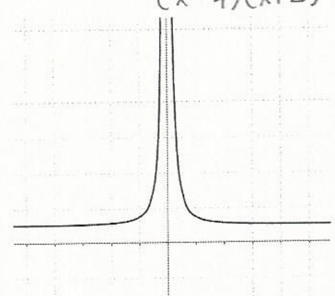
A



D

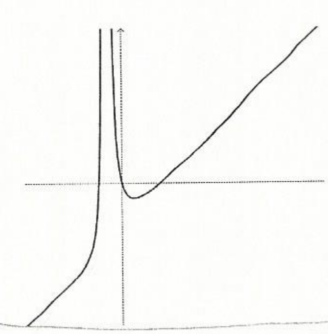


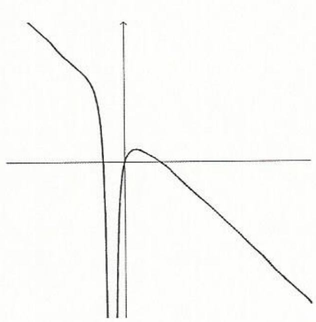
B

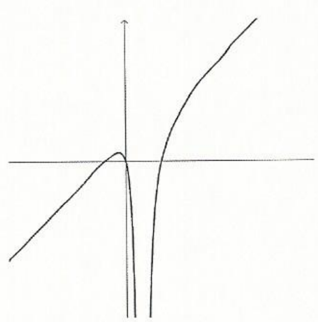


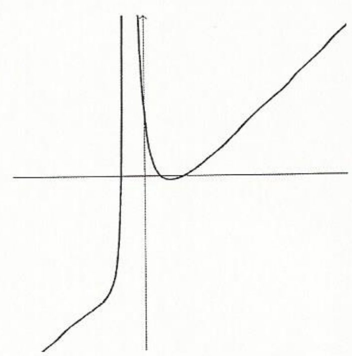
C

17. Which of the four graphs below could be the graph of $f(x)=\dfrac{x^3-2x^2-15x}{x^2+4x+4}$? Explain your choice.









$$\begin{array}{r} x-6 \\ x^2+4x+4\overline{)\,x^3-2x^2-15x+0} \\ -\underline{(x^3+4x^2+4x)} \\ -6x^2-19x+0 \end{array}$$

Find all the **intercepts** of each function.

18. $f(x)=\dfrac{2x+1}{3-x}$

x-intercept: $2x+1=0$
$x=-1/2$

$\boxed{\left(-\frac{1}{2},0\right)}$

y-intercept: $f(0)=\dfrac{2(0)+1}{3-0}=\dfrac{1}{3}$

$\boxed{\left(0,\frac{1}{3}\right)}$

19. $f(x)=\dfrac{x^2-4x+3}{x-2}$

x-intercepts: $x^2-4x+3=0$
$(x-3)(x-1)=0$
$x=3 \quad x=1$

$\boxed{(3,0) \ \& \ (1,0)}$

y-intercept: $f(0)=\dfrac{0-0+3}{0-2}=-\dfrac{3}{2}$

$\boxed{\left(0,-\frac{3}{2}\right)}$

20. $f(x)=\dfrac{x^2-3x-7}{x^2-1}$

x-intercepts: $x^2-3x-7$
$x=\dfrac{3\pm\sqrt{9-4(-7)(1)}}{2}=\dfrac{3\pm\sqrt{37}}{2}$

$\boxed{\left(\frac{3+\sqrt{37}}{2},0\right) \ \& \ \left(\frac{3-\sqrt{37}}{2},0\right)}$

y-intercept: $f(0)=\dfrac{0-0-7}{0-1}=7$ $\boxed{(0,7)}$

21. $f(x)=\dfrac{2x^2+11x-6}{x^3+3x^2-4x+7}$

x-intercepts: $2x^2+11x-6=0$
$(2x-1)(x+6)$
$x=1/2 \quad x=-6$

$\boxed{\left(\frac{1}{2},0\right) \ \& \ (-6,0)}$

y-intercept: $f(0)=\dfrac{0+0-6}{0+0-0+7}=\dfrac{-6}{7}$

$\boxed{\left(0,-\frac{6}{7}\right)}$

Find the **asymptotes and holes** of each function. If the function has a **slant asymptote**, write its equation.

22. $f(x)=\dfrac{x^2+9x+8}{x+2}=\dfrac{(x+1)(x+8)}{x+2}$

Horizontal asymptote:
$\dfrac{x^2}{x}$ → TOP Heavy

$$\begin{array}{r} x+7 \\ x+2\overline{)x^2+9x+8} \\ -x^2+2x \\ \hline 7x+8 \end{array}$$

$\boxed{y=x+7}$

vertical asymptote:
$x+2=0$
$x=-2$

$\boxed{x=-2}$

Holes: none

23. $f(x)=\dfrac{3x-1}{(x+1)(x-3)}$

Horizontal asymptote:
$\dfrac{3x}{x^2}$ → BOTTOM heavy $\boxed{y=0}$

vertical asymptote:
$(x+1)(x-3)=0$
$x=-1 \quad x=3$

$\boxed{x=-1 \ \& \ x=3}$

Holes: none

24. $f(x)=\dfrac{x^2+4x+4}{x^2-2x-15}=\dfrac{(x+2)(x+2)}{(x-5)(x+3)}$

Horizontal asymptote:
$\dfrac{x^2}{x^2}$ → equal → $\dfrac{1}{1}$ $\boxed{y=1}$

vertical asymptote:
$x^2-2x-15=0$
$(x-5)(x+3)=0$
$x=5 \quad x=-3$

$\boxed{x=5,\ x=-3}$

Holes: none

25. $f(x)=\dfrac{2x^3+5x^2-4x-3}{x^2+2}$

Horizontal asymptote:
$\dfrac{2x^3}{x^2}$ → TOP Heavy

$$\begin{array}{r} 2x-8 \\ x^2+0x+2\overline{)2x^3+5x^2-4x-3} \\ 2x^3+0x^2+4x \\ \hline -8x-3 \end{array}$$

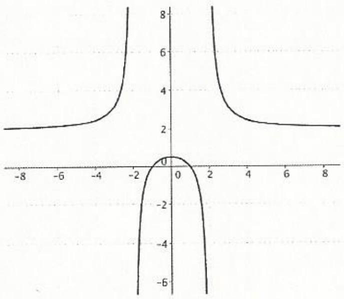
$\boxed{y=2x-8}$

vertical asymptote:
$x^2+2=0$
$x^2=-2$

$\boxed{\text{NO vertical asymptotes}}$

Holes: none

26. Evaluate the limits.




$\lim_{x\to 2^+} f(x) = \underline{\infty}$  $\lim_{x\to 2^-} f(x) = \underline{-\infty}$

$\lim_{x\to -2^+} f(x) = \underline{-\infty}$  $\lim_{x\to -2^-} f(x) = \underline{\infty}$

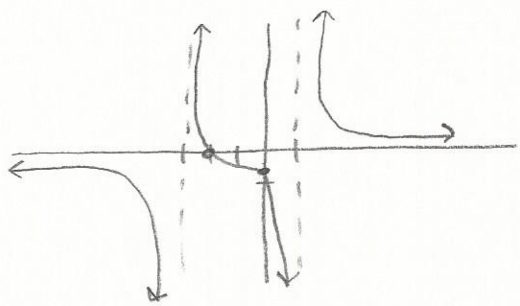
$\lim_{x\to \infty} f(x) = \underline{2}$  $\lim_{x\to -\infty} f(x) = \underline{2}$

Sketch the graph of each function. Clearly show all intercepts and asymptotes of your graph.

27. $f(x) = \dfrac{x+2}{x^2+2x-3} = \dfrac{x+2}{(x+3)(x-1)}$

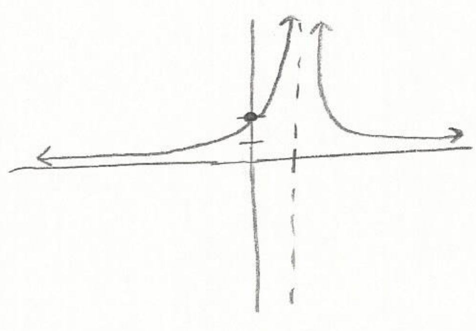
1. H.A.: Bottomheavy $y=0$
2. Holes: None
3. $(x+3)(x-1) \neq 0$  $x=-3$ $x=1$
   $x \neq -3$ $x \neq 1$
4. $x+2=0$  $(-2,0)$
   $x=-2$
5. $f(0) = \dfrac{0+2}{0+0-3} = \dfrac{2}{-3}$  $(0, -\frac{2}{3})$
6. $f(-5) = \dfrac{-5+2}{(-5)^2+2(-5)-3} = \dfrac{-3}{12}$

   $f(2) = \dfrac{2+2}{(2)^2+2(2)-3} = \dfrac{4}{5}$



28. $f(x) = \dfrac{2}{x^2-2x+1} = \dfrac{2}{(x-1)(x-1)}$

1. H.A.: bottom heavy: $y=0$
2. Holes: none
3. $(x-1)(x-1)=0$  $x=1$
   $x=1$
4. $2=0$  $\rightarrow$ no x-intercepts
5. $f(0) = \dfrac{2}{0-0+1} = \dfrac{2}{1}$  $(0,2)$
6. $f(2) = \dfrac{2}{(2)^2-2(2)+1} = \dfrac{2}{1} = 2$



29. $f(x) = \dfrac{x^2-x}{x+1} = \dfrac{x(x-1)}{x+1}$

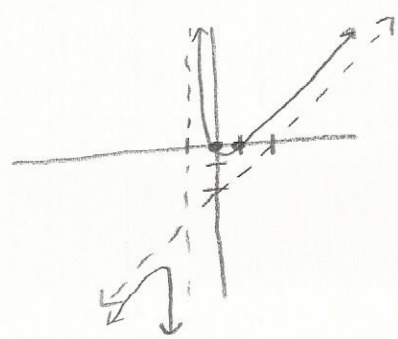
1. H.A.: top heavy  $y = x-2$

$$\begin{array}{r} x-2 \\ x+1 \overline{)\, x^2 - x + 0} \\ -x^2 + -x \\ \hline -2x + 0 \end{array}$$

2. Holes: none
3. $(x+1)=0$  $x=-1$
   $x=-1$
4. $x^2-x=0$
   $x(x-1)=0$  $x=0$  $x=1$  $(0,0)$ $(1,0)$
5. $f(0) = \dfrac{0-0}{0+1} = 0$
   $(0,0)$
6. $f(-2) = \dfrac{(-2)^2-(-2)}{(-2)+1}$
   $= \dfrac{4+2}{-1} = \dfrac{6}{-1} = -6$

   $y = x-2$ at $x=-2$
   $y = -2-2 = -4$

   $-6 < -4$



Solve the rational equations. (Make sure to check for extraneous solutions!)

30. $\left(x-\frac{12}{x}=11\right)x$ LCD: $x$

$x^2-12=11x$

$x^2-11x-12=0$

$(x-12)(x+1)=0$

$x=12 \quad x=-1$

$\boxed{x=12 \text{ and } x=1 \text{ are solutions}}$

31. $\frac{x}{x+2}+\frac{5}{x-3}=\frac{25}{x^2-x-6}$ LCD: $(x-3)(x+2)$

$(x-3)(x+2)$

$x(x-3)+5(x+2)=25$

$x^2-3x+5x+10=25$

$x^2+2x+10=25$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5 \quad x=3$

$\boxed{x=-5 \text{ is a solution. } x=3 \text{ is an extraneous solution.}}$

32. $\frac{2x}{x+2}+\frac{5}{x-5}=\frac{8}{x^2-3x-10}$ LCD: $(x-5)(x+2)$

$(x-5)(x+2)$

$2x(x-5)+5(x+2)=8$

$2x^2-10x+5x+10=8$

$2x^2-5x+2=0$

$(2x-1)(x-2)=0$

$x=2 \quad x=\frac{1}{2}$

$\boxed{x=2 \text{ and } x=\tfrac{1}{2} \text{ are solutions}}$

Solve the polynomial inequality.

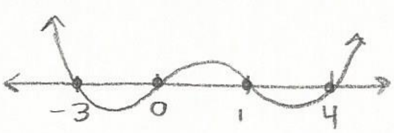
33. $x(x-4)(x-1)(x+3)\geq 0$

degree: 4

l.c.: +

$\uparrow \quad \uparrow$

$x=0 \quad x=4 \quad x=1 \quad x=-3$




$\boxed{(-\infty,-3]\cup[0,1]\cup[4,\infty)}$

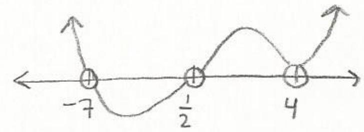
34. $(2x-1)(x-4)^2(x+7)^3<0$

degree: 6

l.c.: +

$\uparrow \quad \uparrow$

$x=\frac{1}{2} \quad x=4 \quad x=-7$



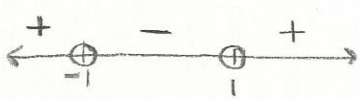

$\boxed{(-7,\tfrac{1}{2})}$

Solve the rational inequality.

35. $\frac{x^2-1}{x^2+4}>0$

zeros: $x=1$, $x=-1$

undefined: none




$\boxed{(-\infty,-1)\cup(1,\infty)}$

36. $\frac{1}{x+1}+\frac{1}{x-3}>0$

$\frac{(x-3)}{(x+1)(x-3)}+\frac{(x+1)}{(x+1)(x-3)}>0$

$\frac{2x-2}{(x+1)(x-3)}>0$

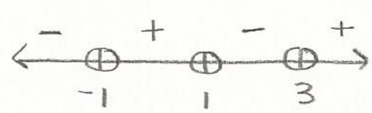
zeros: $2x-2=0$

$2x=2$

$x=1$

undefined: $(x+1)(x-3)=0$

$x=-1 \quad x=3$




$\boxed{(-1,1)\cup(3,\infty)}$

37. $\frac{x^2-5x-6}{x^2+x-12}\leq 0$

zeros: $x^2-5x-6=0$

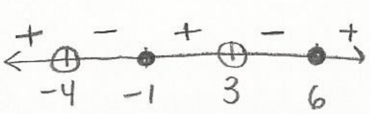
$(x-6)(x+1)=0$

$x=6 \quad x=-1$

undefined: $x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4 \quad x=3$




$\boxed{(-4,-1]\cup(3,6]}$