3.1: Exponential and Logistic Functions

Exponential Functions:

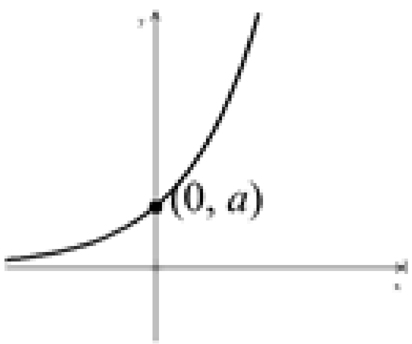
$$f(x) = \underline{a} \cdot \underline{b}^x$$

Let a and b be real number constants. An **exponential function** in x is a function that can be written in the form

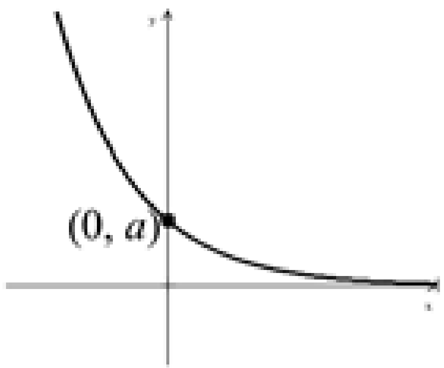
$$f(x) = a \cdot b^x$$

where a is nonzero, b is positive, and $b \neq 1$. The constant a is the ***initial value*** of f (the value at x = 0), and b is the **base.**

$b > 1$



$0 < b < 1$

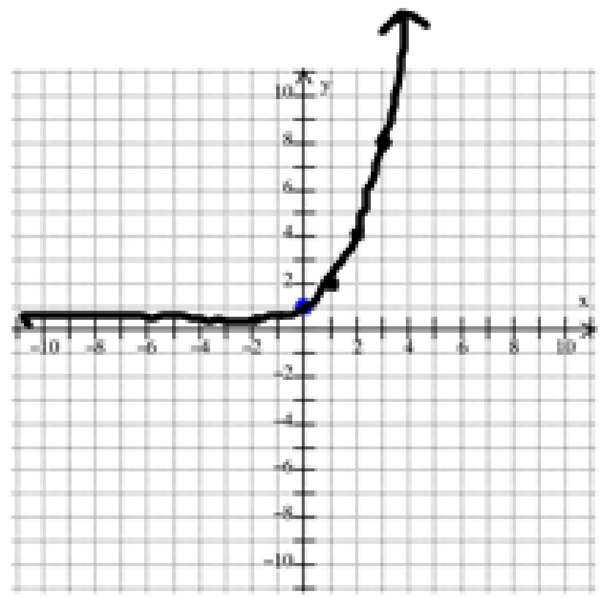


Exponential Tables

Example 1: Create a table of values for $g(x)$ then graph the function.

a) $g(x) = 2^x$

x	$g(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



b) $g(x) = 4 \cdot 3^x$

x	$g(x)$
-2	$\frac{4}{9}$
-1	$\frac{4}{3}$
0	4
1	12
2	36
3	108

$\times 3$

$\times 3$

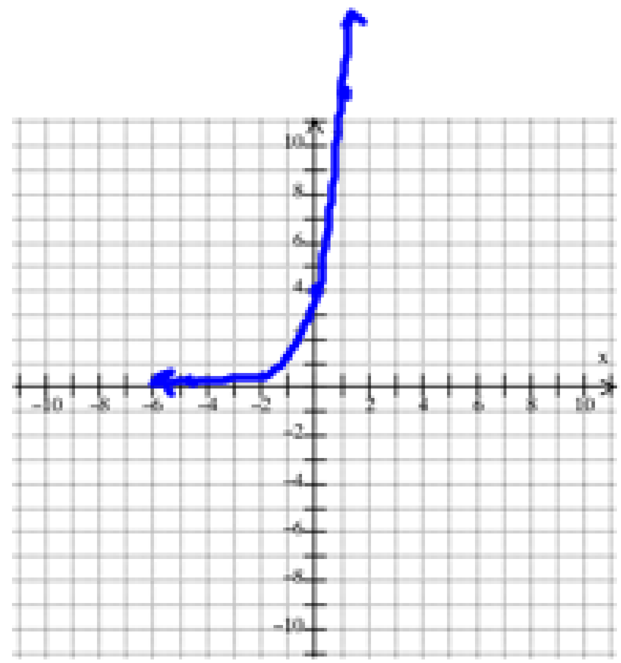
$\times 3$

$\times 3$

$\times 3$

$4 \cdot 3^{-1} = 4\left(\frac{1}{3}\right) = \frac{4}{3}$

$4 \cdot 3^{-2} = 4\left(\frac{1}{9}\right) = \frac{4}{9}$



Finding an Exponential Function from its Table of Values

Example 2: Use the table to determine the formulas for the exponential function.

a)

x	$h(x)$
−2	1.25
−1	2.5
0	5
1	10
2	20

$\times 2$

$\times 2$

$h(x) = 5 \cdot b^x$

$h(0) = 5$

$\boxed{h(x) = 5 \cdot 2^x}$

b)

x	$f(x)$
−2	0.12
−1	0.6
0	3
1	15
2	75

$\boxed{f(x) = 3 \cdot 5^x}$

Exponential Growth and Decay

For any exponential function $f(x) = a \cdot b^x$ *and any real number x,*

$$f(x+1) = b \cdot f(x)$$

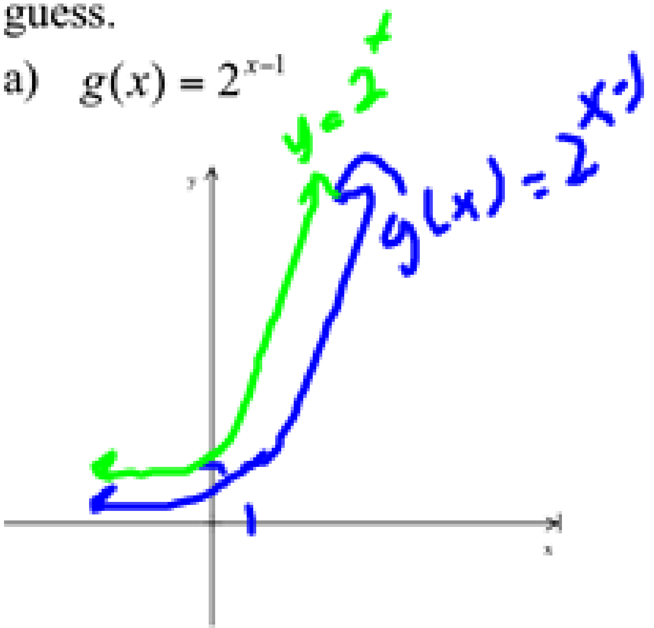
If a > 0 and b >1, *f* is increasing and is an **exponential growth function**.

The base *b* is its **growth factor**.

Transforming Exponential Functions

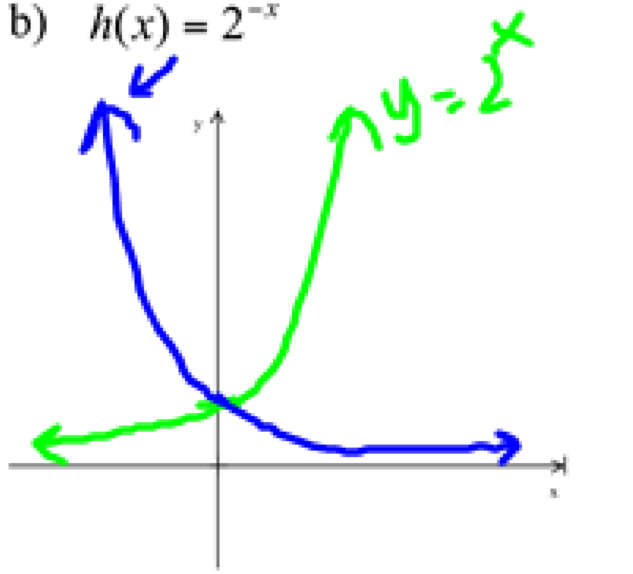
Example 3: Predict what you think the graph will look like. Then graph using a calculator to verify your guess.

a) $g(x) = 2^{x-1}$



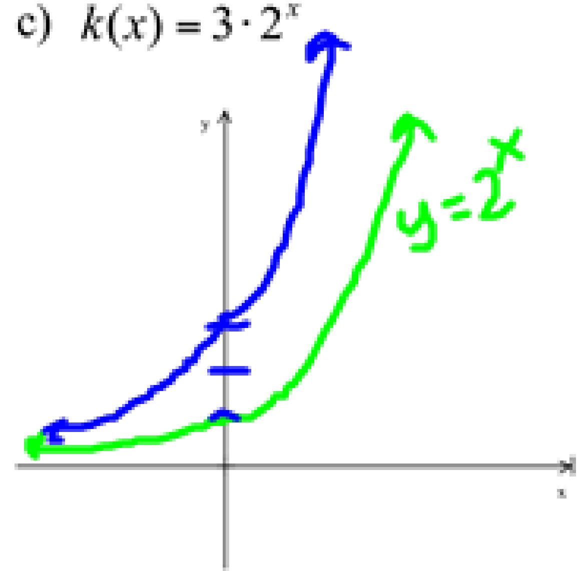
shifted to the right one unit.

b) $h(x) = 2^{-x}$



reflected across the y-axis.

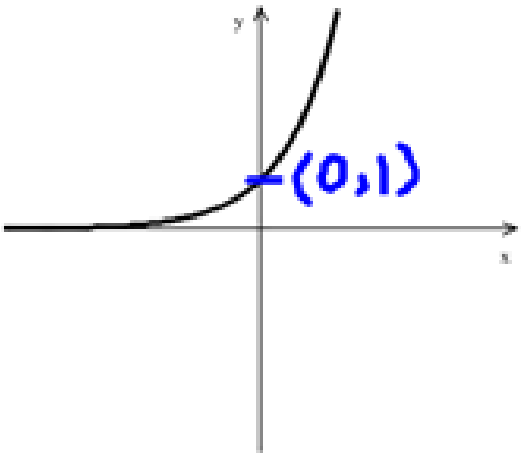
c) $k(x) = 3 \cdot 2^{x}$



stretched by a factor of 3,

The Natural Base *e*

$$f(x) = e^x$$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Continuity: continuous

Increasing: $(-\infty, \infty)$

Decreasing: never

Symmetry: no symmetry : neither odd nor even

Boundedness: Bounded below

Extrema: no local extrema

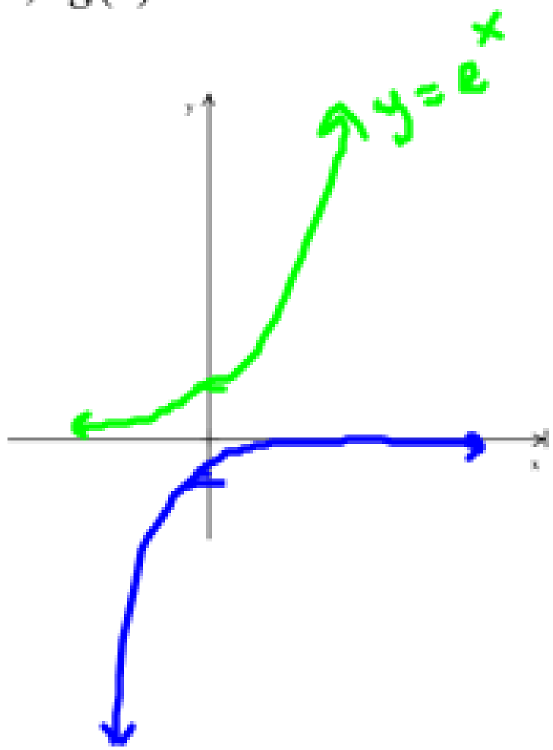
Asymptotes: $y=0$

End Behavior: $\lim_{x \to \infty} f(x) = \infty$

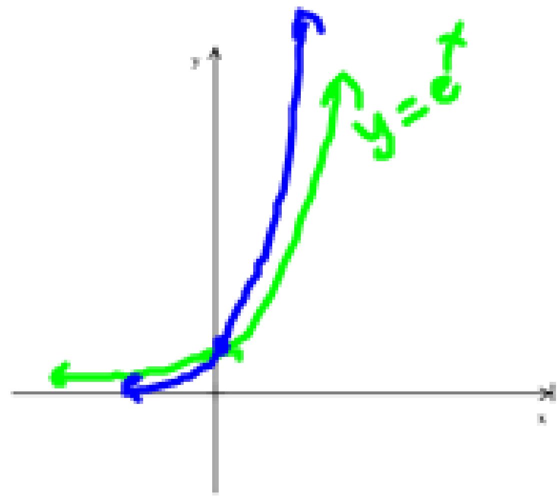
$\lim_{x \to -\infty} f(x) = 0$

Example 4: Predict what you think the graph will look like. Then graph using a calculator to verify your guess.

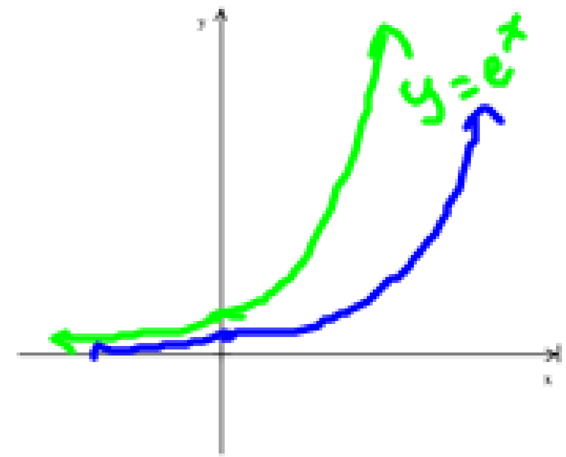
a) $g(x) = -e^{-x}$



b) $h(x) = e^{3x}$



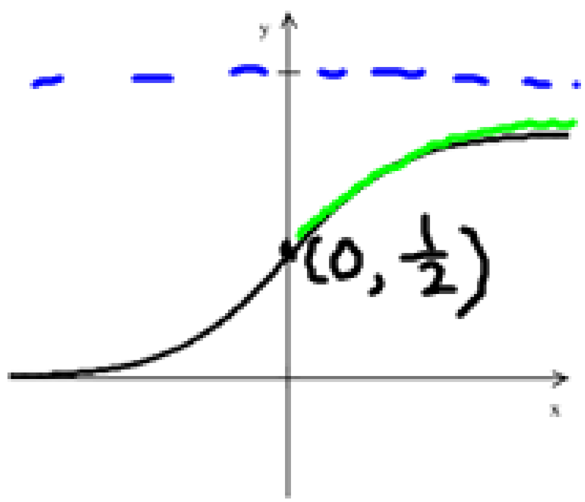
c) $k(x) = \dfrac{1}{4}e^{x}$



pg. 283 $f(x) = \dfrac{c}{1+a e^{-kx}}$ → c is the limit to growth

The Logistic Function

$$f(x) = \frac{\boxed{1}}{1+e^{-x}}$$ → limit to growth



Domain: $(-\infty, \infty)$

Range: $(0, 1)$

Continuity: continuous

Increasing: $(-\infty, \infty)$

Decreasing: never

Symmetry: symmetric about $(0, \frac{1}{2})$

Boundedness: bounded above and below

Extrema: no local extrema

Asymptotes: $y = 0$ $y = 1$

End Behavior: $\lim_{x \to \infty} f(x) = 1$

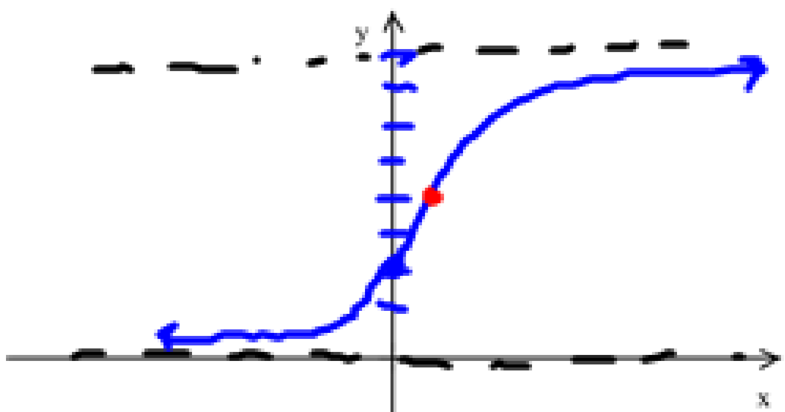
$\lim_{x \to -\infty} f(x) = 0$

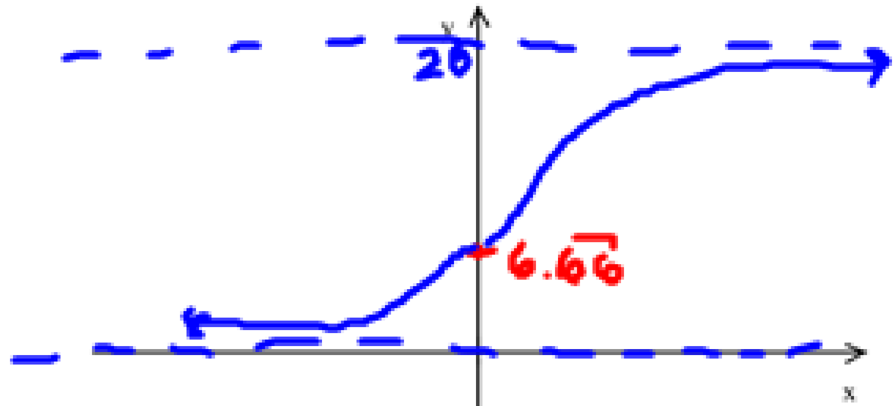
Graphing Logistic Functions

Graph each function using a calculator. Then find the y-intercept and the horizontal asymptotes.

a) $f(x) = \dfrac{8}{1 + 3 \cdot 0.7^{x}}$

b) $g(x) = \dfrac{20}{1 + 2e^{-3x}}$ $e^0 = 1$





$f(0) = \dfrac{8}{1+3(1)} = \dfrac{8}{1+3} = \dfrac{8}{4} = 2$

asymptote: $y=0$ $y=8$

$g(0) = \dfrac{20}{1+2(1)} = \dfrac{20}{3}$

asymptotes: $y=0$ $y=20$