$P(t)=P_0(1\pm r)^t$

3.2: Exponential and Logistic Modeling

Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then

$$P(t) = P_o(1+r)^t,$$

Where P_o is the initial population, r is expressed as a $\boxed{\text{decimal,}}$ and t is time in years.

rate

Example 1: Determine the exponential function with initial value = 12, increasing at a rate of 8% per year.

$P(t)=P_0(1+r)^t$

$P_0=12$

$r=8\%=.08$

$P(t)=12(\underline{1+.08})^t$

$\boxed{P(t)=12(1.08)^t}$

Example 2: Determine the exponential function that satisfies the given conditions.

a) Suppose a culture of 100 bacteria is put into a Petri dish and the culture doubles every hour. Predict when the number of bacteria will be 350,000.

$P(t)=P_0(1+r)^t$

$P_0=100$

$r=100\%=1$

$P(t)=350000$

$\underbrace{350{,}000}_{y_1}=\underbrace{100(2)^t}_{y_2}$

$\boxed{t=11.78 \text{ hours}}$

b) Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially. Find the time when there will be 1 gram of the substance remaining.

$P(t)=P_0(1-r)^t$

$P(t)=1$

$P_0=5$

$r=.5$

$t=\frac{t}{20}$

$\underbrace{1}_{y_1}=\underbrace{5(.5)^{t/20}}_{y_2}$

$t=46.44$

$\boxed{\text{The will be 1 gram on the 47th day}}$