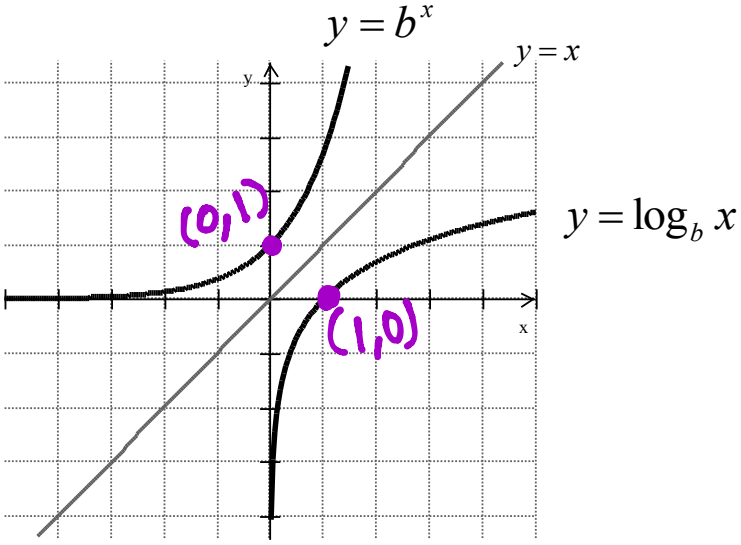
3.3: Logarithmic Functions and Their Graphs

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then $y = \log_b(x)$ if and only if $b^y = x$

Because logarithmic functions are inverse of exponential function we can obtain the graph of the logarithmic function by reflecting the graph over the line $y = x$.



Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y

- $\log_b 1 = 0$ because $b^0 = 1$
- $\log_b b = 1$ because $b^1 = b$
- $\log_b b^y = y$ because $b^y = b^y \Rightarrow y = y$
- $b^{\log_b x} = x$ because $x = x$

Evaluate

a) $\log_3 3 = x$

$3^x = 3^1$

$x = 1$

b) $\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$x = 5$

c) $8^{\log_8 x} = x$

d) $e^{\log_e c} = c$

Common Logarithms-Base 10

Logarithms with base 10 are called common logarithms. y = log x if and only if $10^y = x$

$$y = \log_{10} x \Leftrightarrow y = \log x$$

Basic Properties of Common Logarithms
Let x and y be real numbers with $x > 0$

- $\log_{10} 1 = 0$ because $10^0 = 1$
- $\log_{10} 10 = 1$ because $10^1 = 10$
- $\log_{10} 10^y = y$ because $10^y = 10^y \Rightarrow y = y$
- $10^{\log_{10} x} = x$ because $x = x$

Evaluate.

a) $\log \sqrt[3]{10}$ $\log_{10} 10^{1/3} = \frac{1}{3}$

b) $\log 100$ $\log_{10} 10^2 = 2$

c) $\log \frac{1}{1000} = \frac{1}{10^3}$ $\log_{10} 10^{-3} = -3$

d) $10^{\log 6}$ $= 6$

Natural Logarithms – Base *e*

Basic Properties of Common Logarithms
Let x and y be real numbers with $x > 0$

- $\ln 1 = 0$ because $e^0 = 1$
- $\ln e = 1$ because $1 = 1$ $e^1 = e$
- $\ln e^y = y$ because $y = y$
- $e^{\ln x} = x$ because $x = x$

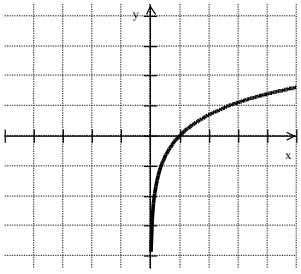
Evaluate.

a) $\ln \frac{1}{e}$ $\ln e^{-1} = -1$

b) $\ln \sqrt[3]{e}$ $\ln e^{1/2} = \frac{1}{2}$

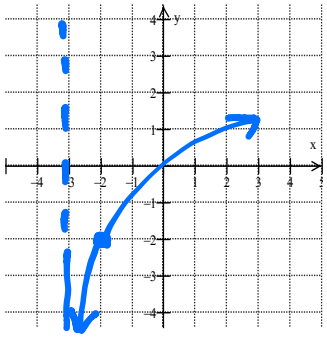
c) $e^{\ln 4} = 4$

$y = \ln x$

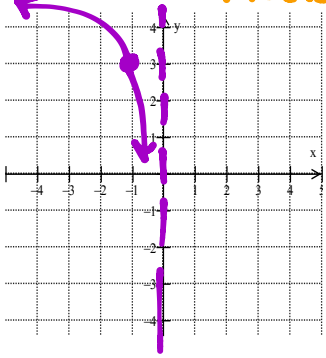


Describe how to transform the graph of $y = \ln x$ into the graph of the given function. Sketch the graph by hand and support your sketch with a calculator.

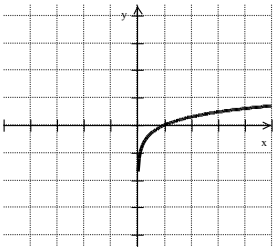
a) $f(x) = \ln(x+3) - 2$ ← down 2, ↑ left 3



b) $g(x) = \ln(-x) + 3$ ← up 3, ↑ reflects across the y

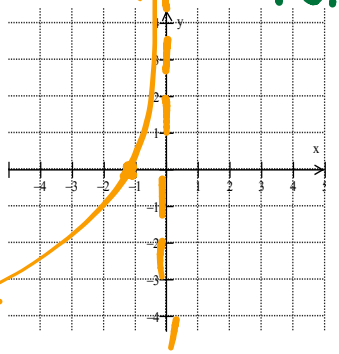


$y = \log x$



Describe how to transform the graph of $y = \log x$ into the graph of the given function. Sketch the graph by hand and support using a calculator.

a) $f(x) = -3\log(-x)$ ← reflects across the x-axis & stretch, ↑ reflects across y-axis



b) $g(x) = 4\log(1-x) + 2$ ← reflect across the y-axis and then to the right 1 unit ($1-x = -(x-1)$), ↑ stretch, ↑ up 2