**3.4 Properties of Logarithmic Functions**

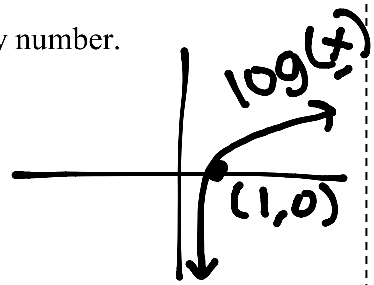
**Properties of Logarithms**

Let x and y be positive numbers and c by any number.

**Product rule:** $\log(x \cdot y) = \log x + \log y$

**Quotient rule:** $\log\left(\dfrac{x}{y}\right) = \log x - \log y$

**Power rule:** $\log x^{c} = c\log x$




Example 1: **Expanding the Logarithm of a Product**
Assuming x and y are positive, use properties of logarithms to write each logarithms as a sum of logarithms or multiples of logarithms.

a) $\log(8xy^4)$

$\log 8 + \log x + \log y^4$

$\boxed{\log 8 + \log x + 4\log y}$

b) $f(x) = \log_{\frac{1}{4}}(16x)$

$\log_{\frac{1}{4}} 16 + \log_{\frac{1}{4}} x$

$\log_{\frac{1}{4}} 16 = z$

$\left(\frac{1}{4}\right)^x = 16$

$4^{-x} = 16$

$4^{-x} = 4^2$

$-x = 2$

$x = -2$

$= \boxed{-2 + \log_{\frac{1}{4}} x}$

c) $f(x) = \log(49x^3y^2)$

$\log_{10} 49 + \log x^3 + \log y^2$

$\boxed{\log 49 + 3\log x + 2\log y}$

Example 2**: Expanding the Logarithm of a Quotient**
Assuming x is positive, the properties of logarithms to write each logarithm as a sum or difference of logarithms or multiples of logarithms.

a) $\ln \frac{\sqrt{x^2+5}}{x}$

$\ln\sqrt{x^2+5} - \ln x$

$\ln(x^2+5)^{1/2} - \ln x$

$\boxed{\frac{1}{2}\ln(x^2+5) - \ln(x)}$

b) $\log \sqrt[4]{\frac{x}{y}} = \frac{\sqrt[4]{x}}{\sqrt[4]{y}}$

$\log\sqrt[4]{x} - \log\sqrt[4]{y}$

$\log x^{\frac{1}{4}} - \log y^{\frac{1}{4}}$

$\boxed{\frac{1}{4}\log x - \frac{1}{4}\log y}$

$\frac{1}{4}(\log x - \log y)$

Example 3: **Condensing a logarithmic Expression**
Assuming x an y are positive, write each logarithm as a single logarithm

a) $\ln x^5 - 2\ln(xy)$

$\ln x^5 - \ln(xy)^2$

$\ln\left(\frac{x^5}{(xy)^2}\right) = \ln\left(\frac{x^5}{x^2y^2}\right)$

$= \boxed{\ln\left(\frac{x^3}{y^2}\right)}$

b) $\log x - \underbrace{\log 10}_{1}$

$\boxed{\log x - 1}$

$\log_{10} 10 = y$

$10^y = 10^1$

**Change -of-Base Formula for Logarithms**
For positive real numbers a, b, and x with a ≠ 1 and b ≠ 1

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Example 5: **Evaluating Logarithms by Changing the Base**

a) $\log_3 16 =$

$\frac{\log 16}{\log 3}$ $\quad \frac{\ln 16}{\ln 3}$

$= \log(16)/\log(3)$

$\approx \boxed{2.524}$

b) $\log_6 10 =$

$\frac{\log 10}{\log 6} = \frac{1}{\log 6}$

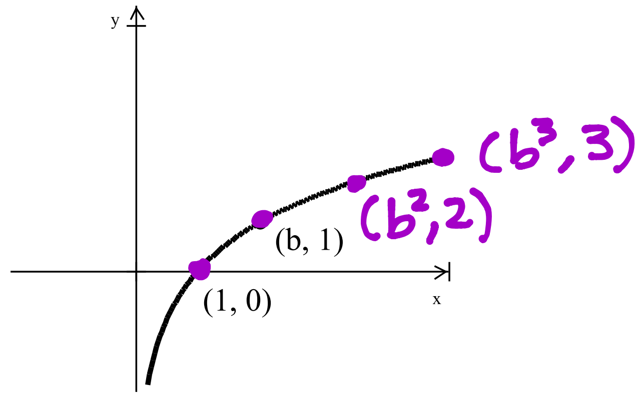
$1/\log(6)$

$\approx \boxed{1.285}$

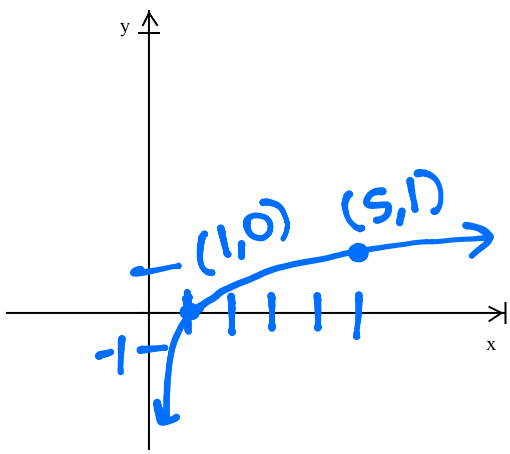
c) $\log_{\frac{1}{2}} 2 =$

$\frac{\log 2}{\log \frac{1}{2}} = \boxed{-1}$

**Graphing Logarithmic Functions** $f(x) = \log_b x$




a). $g(x) = \log_5 x$




b). $h(x) = \log_{\frac{1}{4}} x$