

Precalculus

4.1-4.3 Review

Do your work ON A SEPARATE SHEET OF PAPER.

Convert from radians to degrees.

1. $\frac{2\pi}{3}$

2. $\frac{\pi}{4}$

$$\frac{2\pi}{3} \left(\frac{180}{\pi} \right) = \boxed{120^\circ}$$

$$\frac{\pi}{4} \left(\frac{180}{\pi} \right) = \boxed{45^\circ}$$

Convert from degrees to radians.


3. 150°

4. 270°

$$150 \left(\frac{\pi}{180} \right) = \boxed{\frac{5\pi}{6}}$$


$$270 \left(\frac{\pi}{180} \right) = \boxed{\frac{3\pi}{2}}$$

5. a. If the measure of a sector is 60° and has a radius of 5 inches, find the arc length.



$$\sin \left(\frac{60^\circ}{180^\circ} \right) (2\pi r) = (10\pi) \left(\frac{60}{360} \right) = \boxed{\frac{5\pi}{3} \text{ inches}}$$

b. If the arc length of a sector is 2.5 cm and has an angle measure of $\frac{\pi}{3}$ radians, find the radius.



$$\text{arc length: } (2\pi r) \left(\frac{\theta}{2\pi} \right)$$

$$2.5 = (2\pi r) \left(\frac{\frac{\pi}{3}}{2\pi} \right) \Rightarrow 3 \cdot 2.5 = \frac{\pi}{3} r \cdot 3$$

$$\frac{7.5}{\pi} = \frac{\pi r}{\pi}$$

$$\boxed{\text{radius} = \frac{7.5}{\pi} \text{ cm}}$$

6. A Ferris wheel with radius 60 feet makes one complete turn every 40 seconds.

- What its **linear speed** (in feet per second)?

$$\left(\frac{1 \text{ rev}}{40 \text{ sec}} \right) \left(\frac{120\pi \text{ ft}}{1 \text{ rev}} \right) = \boxed{3\pi \text{ feet per second}}$$

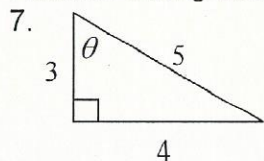
- What is its **angular speed** in degrees per second?

$$\frac{360^\circ}{40 \text{ sec}} = \boxed{9^\circ \text{ per second}}$$

- What is its **angular speed** in radians per second?

$$\frac{2\pi}{40 \text{ sec}} = \boxed{\frac{\pi}{20} \text{ radians per second}}$$

Find the six trig functions of the angle θ .



$$\sin \theta = \frac{4}{5} \quad \csc \theta = \frac{5}{4}$$

$$\cos \theta = \frac{3}{5} \quad \sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{4}{3} \quad \cot \theta = \frac{3}{4}$$

Evaluate without using a calculator. (You must have these memorized).

8. $\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ 9. $\cos \frac{\pi}{3} = \frac{1}{2}$ 10. $\tan \frac{\pi}{4} = 1$ 11. $\cos 30^\circ = \frac{\sqrt{3}}{2}$ 12. $\sin \frac{\pi}{6} = \frac{1}{2}$

13. $\tan \frac{\pi}{3} = \sqrt{3}$ 14. $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ 15. $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ 16. $\sec \frac{\pi}{4} = \frac{1}{\cos \frac{\pi}{4}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$ 17. $\cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

Find the acute angle θ that satisfies the given equation.

* Recall: an acute angle is less than 90°

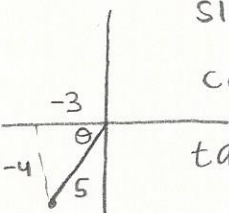
18. $\cos \theta = \frac{\sqrt{3}}{2}$ 19. $\sin \theta = \frac{\sqrt{2}}{2}$ 20. $\csc \theta = 2$ 21. $\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or 30° $\theta = 45^\circ$ or $\frac{\pi}{4}$ $\sin \theta = \frac{1}{2}$ $\theta = 30^\circ$ or $\frac{\pi}{6}$

$\theta = \frac{\pi}{6}$ or 30°

Point P is on the terminal side of angle θ . Evaluate the six trig functions.

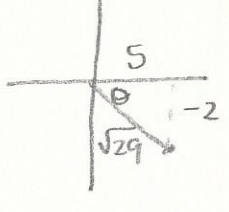
22. $(-3, -4)$ 23. $(5, -2)$



$\sin \theta = \frac{-4}{5}$ $\csc \theta = \frac{-5}{4}$ $\sin \theta = \frac{-2}{\sqrt{29}}$ $\csc \theta = \frac{-\sqrt{29}}{2}$

$\cos \theta = \frac{-3}{5}$ $\sec \theta = \frac{-5}{3}$ $\cos \theta = \frac{5}{\sqrt{29}}$ $\sec \theta = \frac{\sqrt{29}}{5}$

$\tan \theta = \frac{-4}{-3} = \frac{4}{3}$ $\cot \theta = \frac{3}{4}$ $\tan \theta = \frac{-2}{5}$ $\cot \theta = \frac{-5}{2}$



Name two positive coterminal angles and two negative coterminal angles to the given angle.

24. 55° 25. -320° 26. $\frac{5}{6}\pi$

Positive: $55 + 360 = 415^\circ$ Pos.: $-320 + 360 = 40^\circ$ Pos.: $\frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$

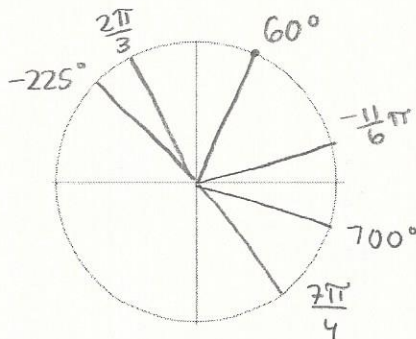
$415 + 360 = 775^\circ$ $40 + 360 = 400^\circ$ $\frac{17\pi}{6} + 2\pi = \frac{29\pi}{6}$

Negative: $55 - 360 = -305^\circ$ Neg.: $-320 - 360 = -680^\circ$ Neg.: $\frac{5\pi}{6} - 2\pi = \frac{-7\pi}{6}$

$-305 - 360 = -665^\circ$ $-680 - 360 = -1040^\circ$ $\frac{-7\pi}{6} - 2\pi = \frac{-19\pi}{6}$

Locate the angle on the unit circle.

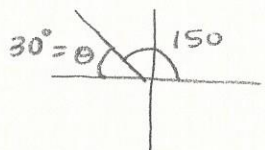
- 27. 60°
- 28. $700^\circ = 360 + 340^\circ$
- 29. -225°
- 30. $\frac{2\pi}{3}$
- 31. $\frac{7\pi}{4}$
- 32. $-\frac{11\pi}{6}$



S	A
T	C

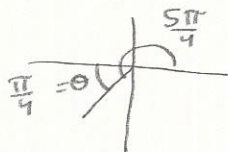
Evaluate without using a calculator.

33. $\sin 150^\circ$



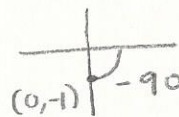
$$\sin 150 = \sin 30 = \frac{1}{2}$$

34. $\cos \frac{5\pi}{4}$



$$\cos \frac{5\pi}{4} = -\cos \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

35. $\tan(-90^\circ)$



$$\tan = \frac{y}{x}$$

$$\tan(-90) = \frac{-1}{0}$$

undefined

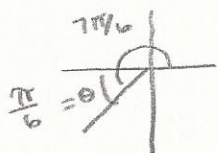
36. $\sin\left(-\frac{2\pi}{3}\right)$



$$\sin\left(-\frac{2\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

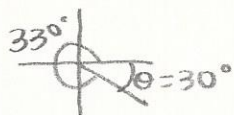
37. $\cot\left(\frac{7\pi}{6}\right) = \frac{1}{\tan\left(\frac{7\pi}{6}\right)}$



$$\cot\left(\frac{7\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

38. $\sec 330^\circ = \frac{1}{\cos 330}$



$$\sec(330) = \frac{1}{\cos 30} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

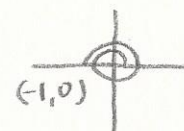
39. $\cos 135^\circ$



$$\cos 135^\circ = -\cos 45^\circ$$

$$= -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

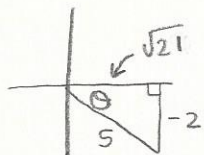
40. $\csc(3\pi) = \frac{1}{\sin 3\pi}$



$$\csc(3\pi) = \frac{1}{\sin 3\pi} = \frac{1}{0}$$

undefined.

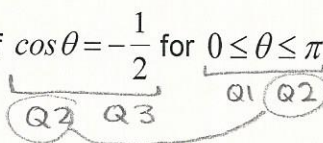
41. Find $\tan \theta$ and $\sec \theta$ if $\sin \theta = -\frac{2}{5}$ and $\cos \theta > 0$.



$$\tan \theta = \frac{-2}{\sqrt{21}} \text{ or } \frac{-2\sqrt{21}}{21}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{\sqrt{21}}{5}} = \frac{5}{\sqrt{21}} \text{ or } \frac{5\sqrt{21}}{21}$$

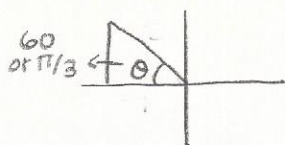
42. If $\cos \theta = -\frac{1}{2}$ for $0 \leq \theta \leq \pi$, what is the measure of θ ?



Recall: $\cos 60 = \frac{1}{2}$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$$

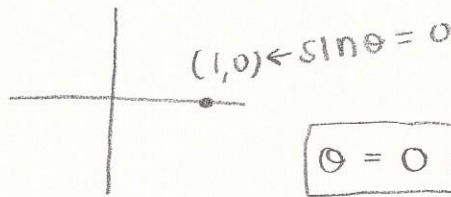


Since this is a measure in radians, your answer should be in radians.

43. If $\tan \theta = 0$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, what is the measure of θ ? ← Q4 and Q1

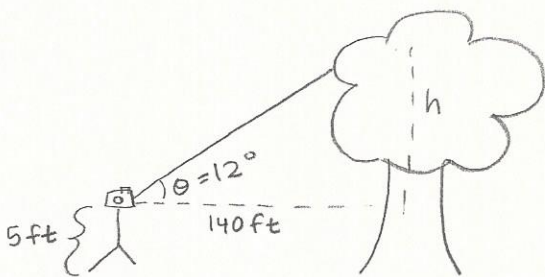
$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{0}{\neq}$$

$$\sin \theta = 0$$



Your answer cannot be 2π because θ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. 2π is not in that interval.

44. Kristen places her surveyor's telescope on the top of a tripod 5 feet above the ground. She measures an 12° elevation above the horizontal to the top of a tree that is 140 feet away. How tall is the tree? Round your answer to the nearest hundredth foot.



$$140 \cdot \tan 12^\circ = \frac{h}{140} \cdot 140$$

$$h = 140 \tan 12^\circ$$

$$h = 29.76$$

But, the camera is 5 feet above the ground.

$$\text{Height of the tree: } 29.76 + 5 = \boxed{34.76 \text{ feet}}$$

45. A radial arm saw has a circular cutting blade with a diameter of 14 inches. It spins 1500 rpm. If there are 10 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?

$$\left(\frac{1500 \text{ rev}}{1 \text{ min}}\right) \left(\frac{14 \pi \text{ in}}{1 \text{ rev}}\right) \left(\frac{10 \text{ teeth}}{1 \text{ in}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \boxed{\begin{array}{l} 3500 \pi \text{ teeth per second} \\ \approx 10995.57 \text{ teeth per second} \end{array}}$$