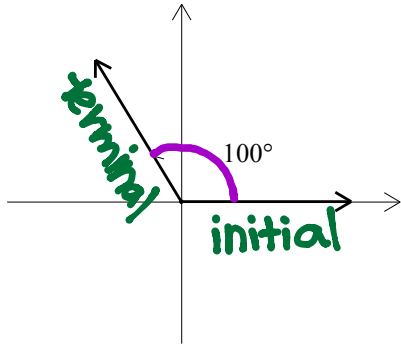


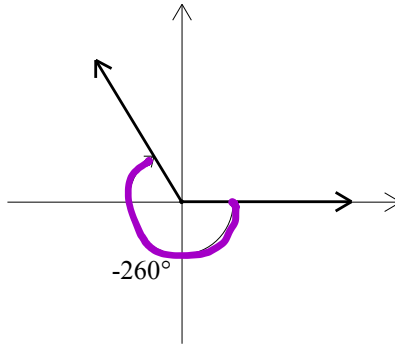
4.3: Trigonometry Extended: The Circular Functions

Coterminal Angles: Angles that have the same initial and terminal side.

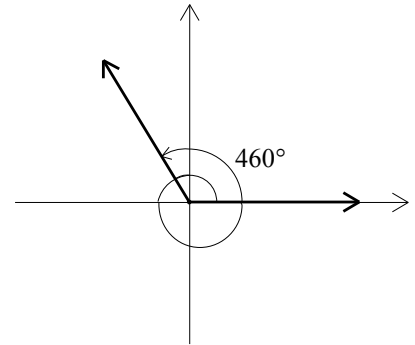
All three angles below are coterminal.



Positive coterminal angle



Negative coterminal angle



Positive coterminal angle

Example 1: Finding Coterminal Angles

Find and draw a positive angle and a negative angle that are coterminal with the given angle.

a) 30°

positive: $30 + 360 = 390^\circ$

negative: $30 - 360 = -330^\circ$

b) -150°

positive: $-150 + 360 = 210^\circ$

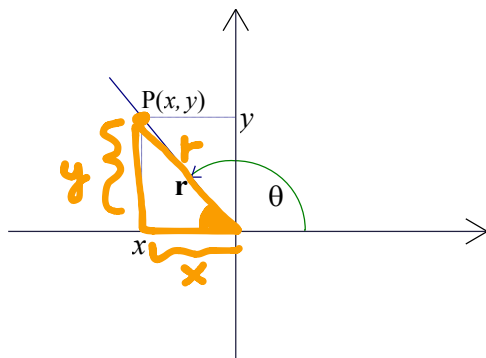
negative: $-150 - 360 = -510^\circ$

c) $\frac{2\pi}{3}$

positive: $\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

negative: $\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

Trigonometric Functions of any Angle



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc \theta = \frac{r}{y} \quad (y \neq 0)$$

$$\sec \theta = \frac{r}{x} \quad (x \neq 0)$$

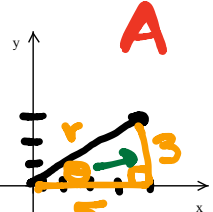
$$\cot \theta = \frac{x}{y} \quad (y \neq 0)$$

Example 2: Evaluating Trig Functions Determined by a Point

Let θ be the acute angle in standard position whose terminal side contains the given point. Find the six trigonometric functions of θ .

a) (5, 3)

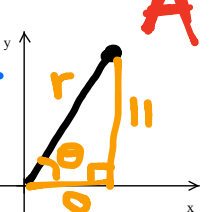
$3^2 + 5^2 = r^2$
 $9 + 25 = r^2$
 $34 = r^2$
 $r = \sqrt{34}$



$\sin \theta = \frac{3}{\sqrt{34}}$ $\csc \theta = \frac{\sqrt{34}}{3}$
 $\cos \theta = \frac{5}{\sqrt{34}}$ $\sec \theta = \frac{\sqrt{34}}{5}$
 $\tan \theta = \frac{3}{5}$ $\cot \theta = \frac{5}{3}$

b) (8, 11)

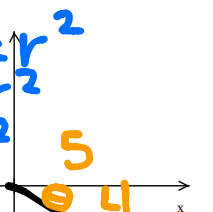
$8^2 + 11^2 = r^2$
 $64 + 121 = r^2$
 $185 = r^2$
 $r = \sqrt{185}$



$\sin \theta = \frac{11}{\sqrt{185}}$ $\csc \theta = \frac{\sqrt{185}}{11}$
 $\cos \theta = \frac{8}{\sqrt{185}}$ $\sec \theta = \frac{\sqrt{185}}{8}$
 $\tan \theta = \frac{11}{8}$ $\cot \theta = \frac{8}{11}$

c) (5, -3)

$5^2 + (-3)^2 = r^2$
 $25 + 9 = r^2$
 $34 = r^2$
 $r = \sqrt{34}$

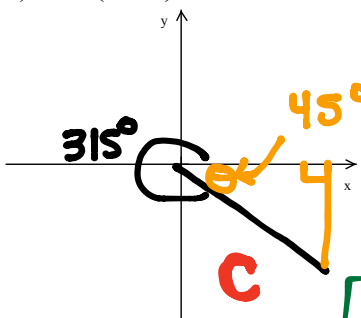


$\sin \theta = \frac{-3}{\sqrt{34}}$ $\csc \theta = \frac{\sqrt{34}}{-3}$
 $\cos \theta = \frac{5}{\sqrt{34}}$ $\sec \theta = \frac{\sqrt{34}}{5}$
 $\tan \theta = \frac{-3}{5}$ $\cot \theta = \frac{5}{-3}$

Example 3: Evaluating Trig Functions

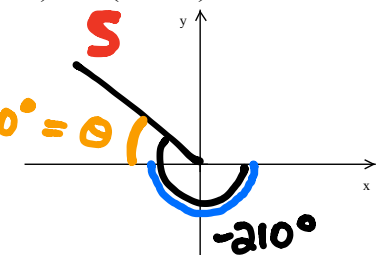
Find the following without using a calculator.

a) $\cos(315^\circ)$



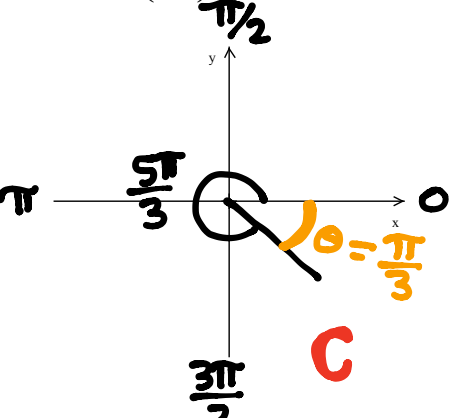
$\cos(315^\circ) = \cos(45^\circ) = \frac{1}{\sqrt{2}}$

b) $\sin(-210^\circ)$



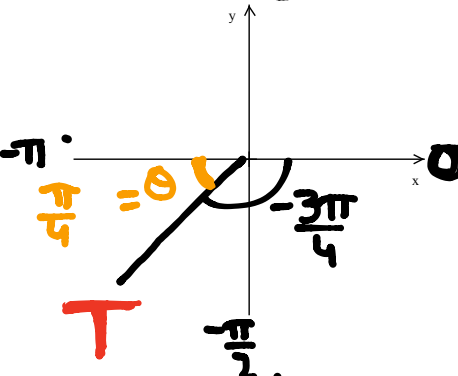
$\sin(-210^\circ) = \sin(30^\circ) = \frac{1}{2}$

c) $\tan\left(\frac{5\pi}{3}\right)$



$\tan\left(\frac{5\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

d) $\sec\left(-\frac{3\pi}{4}\right) = \frac{1}{\cos\left(-\frac{3\pi}{4}\right)}$

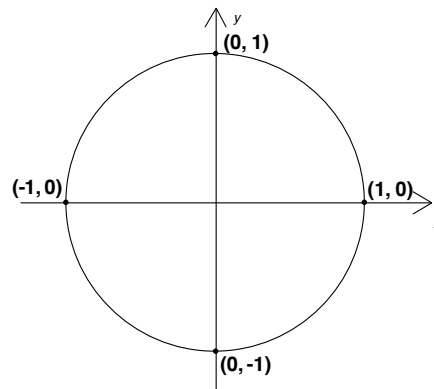
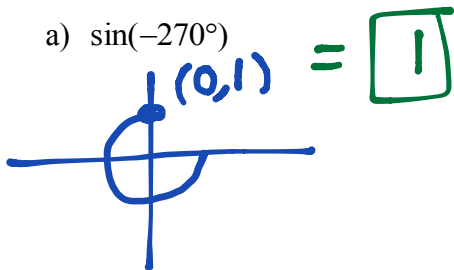


$\cos\left(-\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$
 $\sec\left(-\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\sqrt{2}$

Example 4: Evaluating Trig Functions of Quadrantal Angles

Find each of the following, if it exists. If the value is undefined write "undefined".

a) $\sin(-270^\circ)$

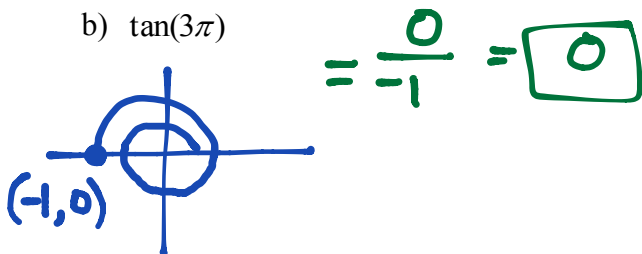


Unit Circle

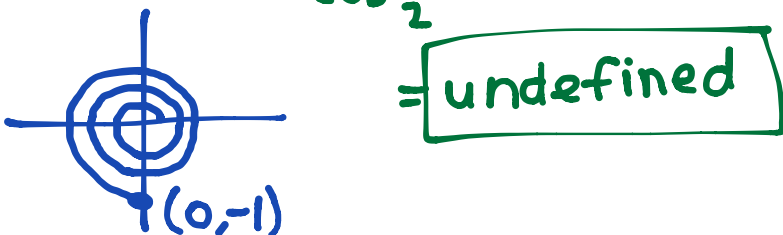
$$(x, y) = (\cos\theta, \sin\theta)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

b) $\tan(3\pi)$



c) $\sec \frac{11\pi}{2} = \frac{1}{\cos \frac{11\pi}{2}} = \frac{1}{0}$



Example 5: Using one trig ratio to find the others

Find $\cos\theta$ and $\tan\theta$ by using the given information to construct a reference triangle.

a) $\sin\theta = \frac{3}{7}$ and $\tan\theta < 0$

b) $\sec\theta = 3$ and $\sin\theta > 0$



$$\begin{aligned} x^2 + 3^2 &= 7^2 \\ x^2 + 9 &= 49 \\ x^2 &= 40 \\ x &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} \cos\theta &= \frac{-2\sqrt{10}}{7} \\ \tan\theta &= \frac{3}{-2\sqrt{10}} \end{aligned}$$



$$\cos\theta = \frac{1}{3}$$

$$\begin{aligned} 3^2 &= 1^2 + y^2 \\ 9 &= 1 + y^2 \\ 8 &= y^2 \\ y &= 2\sqrt{2} \end{aligned}$$

$$\tan\theta = \frac{2\sqrt{2}}{1}$$