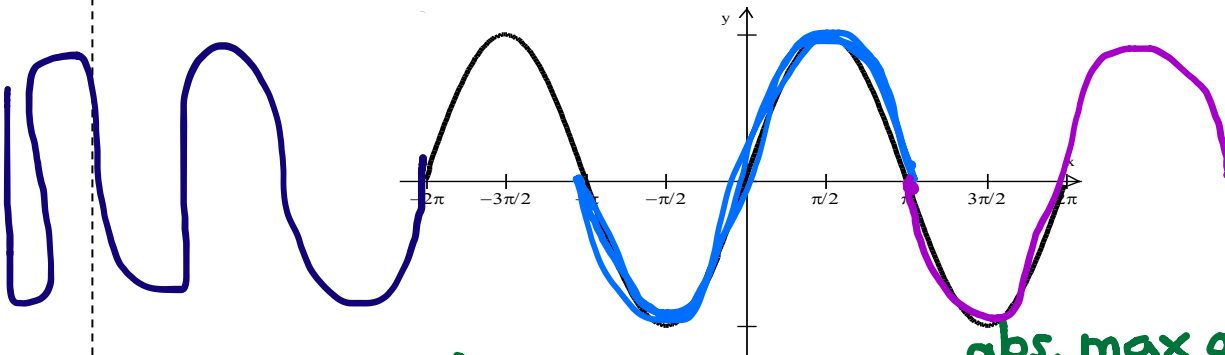


4.4: Graphs of Sine and Cosine: Sinusoids

The Sine Function

$$f(x) = \sin x$$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Continuity: **continuous**

Symmetry: **odd** (symmetric about the origin)

Boundedness: **Bounded**

Extrema: **abs. max at 1**
abs. min at -1

Asymptotes: **none**

End Behavior:

$$\lim_{x \rightarrow -\infty} \sin x = \text{DNE}$$

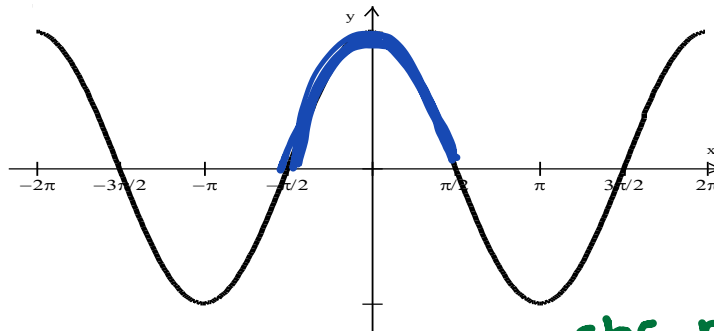
$$\lim_{x \rightarrow \infty} \sin x = \text{DNE}$$

oscillating between -1 and 1

Periodic with period 2π

The Cosine Function

$$f(x) = \cos x$$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Continuity: **continuous**

Symmetry: **even** (symmetric about the y-axis)

Boundedness: **Bounded**

Extrema: **abs. max at 1**
abs. min at -1

Asymptotes: **NONE**

End Behavior:

$$\lim_{x \rightarrow -\infty} \cos x = \text{DNE}$$

$$\lim_{x \rightarrow \infty} \cos x = \text{DNE}$$

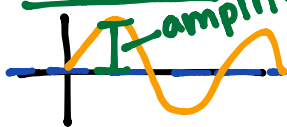
oscillating between -1 and 1

Periodic with period 2π

Transformations of Sine and Cosine Graphs

$f(x) = a \sin(b(x-c)) + d$ or $f(x) = a \cos(b(x-c)) + d$

$|a|$ = amplitude b effects the period and frequency of the graph,



$\text{new period} = \frac{2\pi}{|b|}$

 $\text{frequency} = \frac{|b|}{2\pi}$

c = horizontal shift d = vertical shift

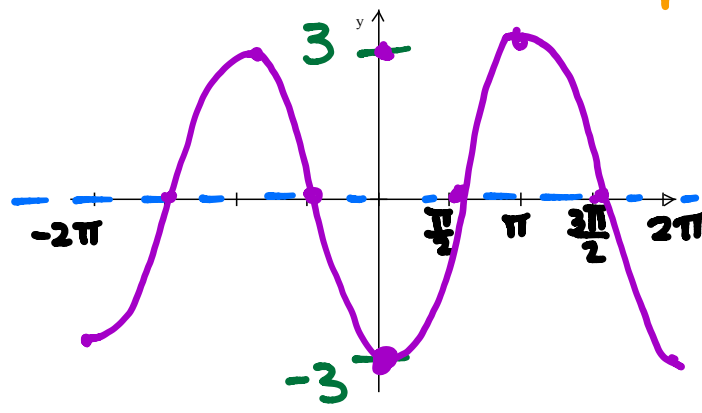
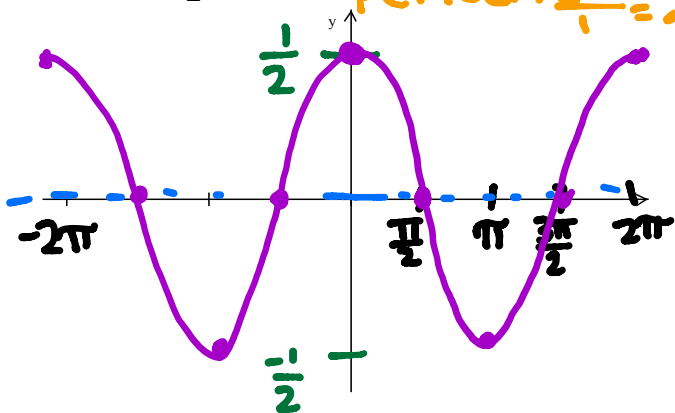
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Example 1: Changing Amplitude

Find the amplitude of each function and use it to sketch the graph.

a) $f(x) = \frac{1}{2} \cos x$ amplitude: $\frac{1}{2}$
period: $\frac{2\pi}{1} = 2\pi$

b) $g(x) = -3 \cos x$ amplitude: 3
period: $\frac{2\pi}{1} = 2\pi$

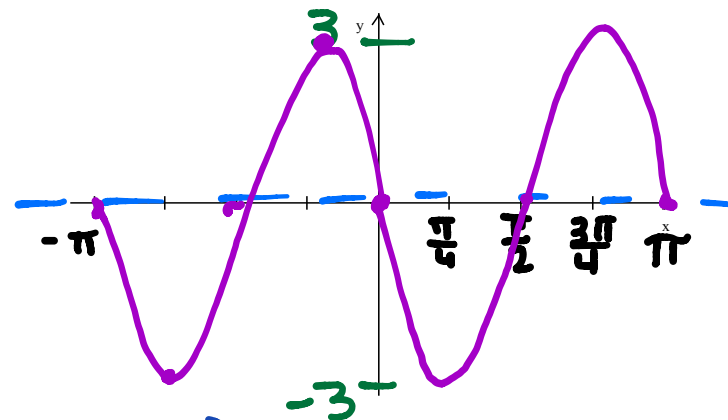
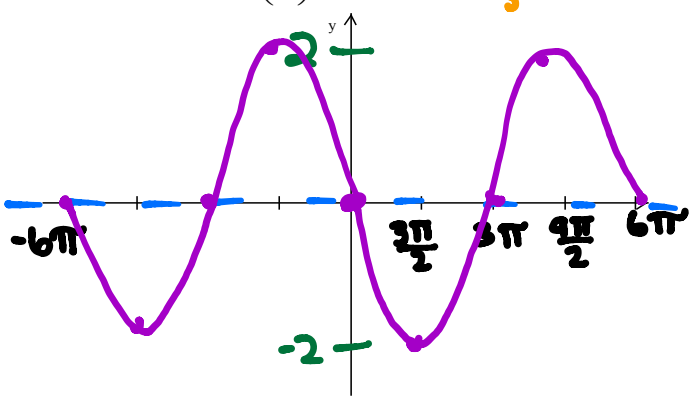


Example 2: Horizontal Stretch or Shrink and Period Change

Find the period of each function and use the language of transformations to describe how the graphs are related.

a) $f(x) = -2 \sin\left(\frac{x}{3}\right)$ amplitude: 2
period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$

b) $g(x) = 3 \sin(-2x)$ amplitude: 3
period: $\frac{2\pi}{2} = \pi$



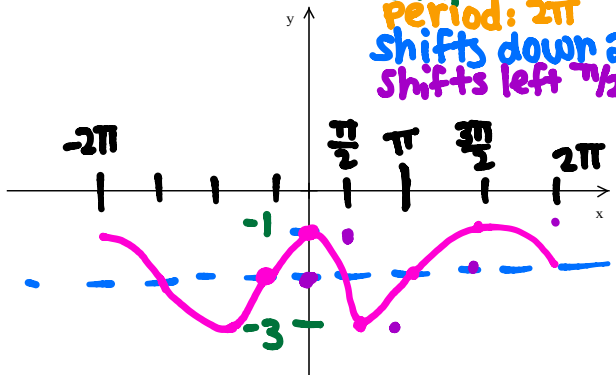
$$-f(x) = f(-x)$$

Example 3: Graphing Transformations

Graph two periods of each sine or cosine function.

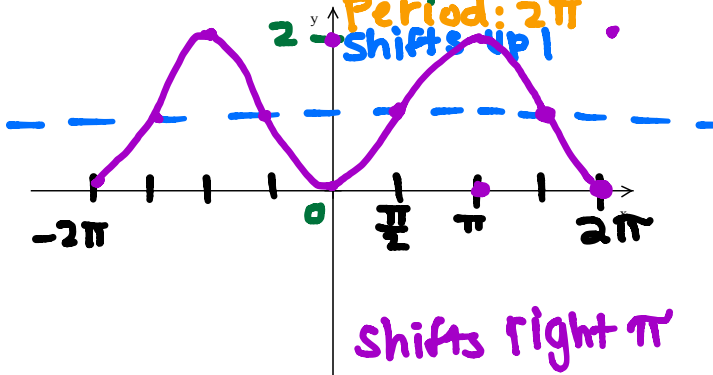
a) $f(x) = \sin\left(x + \frac{\pi}{2}\right) - 2 = \sin\left(1\left(x + \frac{\pi}{2}\right)\right) - 2$

amplitude: 1
 Period: 2π
 shifts down 2
 shifts left $\pi/2$



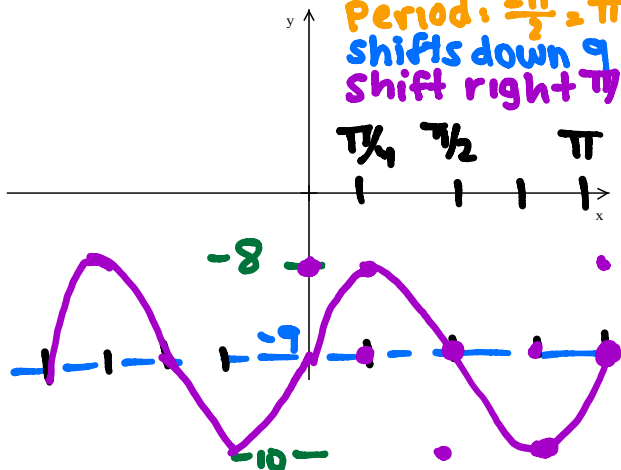
b) $f(x) = \cos(x - \pi) + 1 = \cos(1(x - \pi)) + 1$

amplitude: 1
 Period: 2π
 shifts up 1



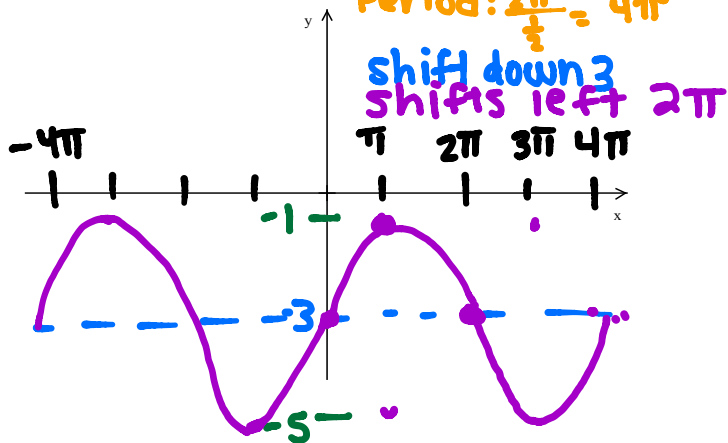
c) $f(x) = \cos\left(2x - \frac{\pi}{4}\right) - 9 = \cos\left(2\left(x - \frac{\pi}{8}\right)\right) - 9$

amplitude: 1
 Period: $\frac{2\pi}{2} = \pi$
 shifts down 9
 shift right $\pi/4$



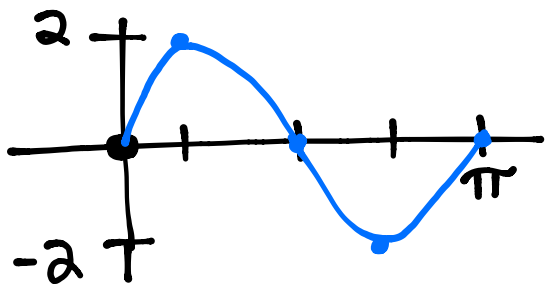
d) $f(x) = -2\sin\left(\frac{x}{2} + \pi\right) - 3 = -2\sin\left(\frac{1}{2}\left(x + 2\pi\right)\right) - 3$

amplitude: 2
 Period: $\frac{2\pi}{1/2} = 4\pi$
 shift down 3
 shifts left 2π



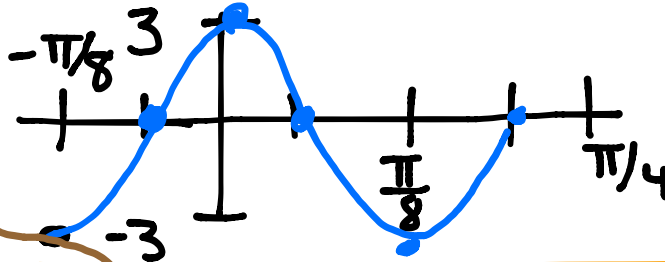
Example 5: Writing equations of Sine and Cosine graphs

a) amplitude = 2, period = π , point (0, 0)



$y = 2\sin(2x)$

b) amplitude = 3, period = $\frac{\pi}{4}$, point $\left(-\frac{\pi}{8}, -3\right)$



$y = 3\cos(8x)$

$\frac{2\pi}{b} = \frac{\pi}{4}$
 $8\pi = b\pi$
 $b = 8$

$\frac{2\pi}{b} = \pi$
 $\frac{2\pi}{\pi} = \frac{b\pi}{\pi}$
 $b = 2$