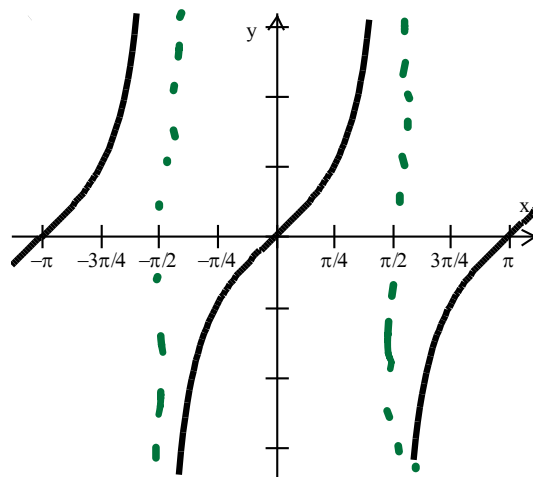


4.5: Graphs of Tangent, Cotangent, Secant, and Cosecant

The Tangent Function

$$f(x) = \tan x$$



Domain: All reals except odd multiples of $\frac{\pi}{2}$

$$\left(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \right)$$

Range: $(-\infty, \infty)$

Continuous on its domain

Increasing on each interval of the domain

Symmetric with respect to the origin (odd)

Not bounded above or below

No local extrema

No horizontal asymptotes

Vertical asymptotes: $x = k \cdot \frac{\pi}{2}$ for all odd integers k

End Behavior: $\lim_{x \rightarrow -\infty} \tan x$ and $\lim_{x \rightarrow \infty} \tan x$
Do Not Exist (DNE)

Periodic with period π

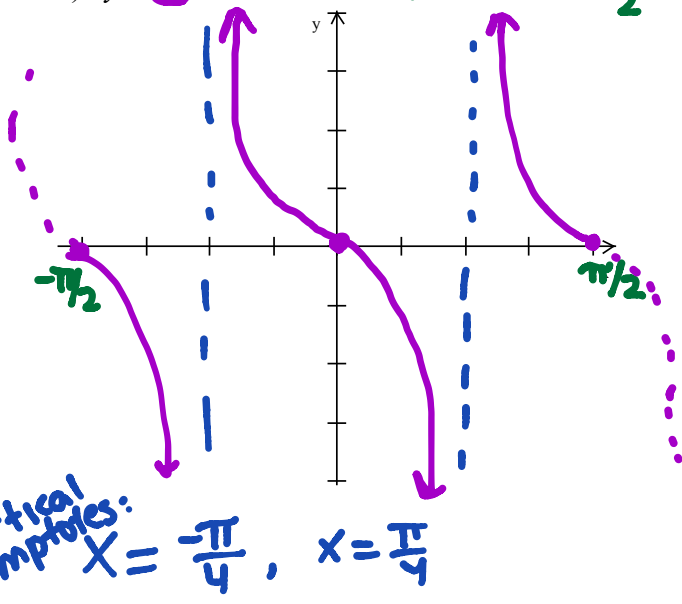
Since $\tan x = \frac{\sin x}{\cos x}$, $\tan x$ has vertical asymptotes everywhere $\cos x = 0$

Example 1: Graphing Tangent Functions

Locate the vertical asymptotes and graph two periods of the function. Describe the transformations.

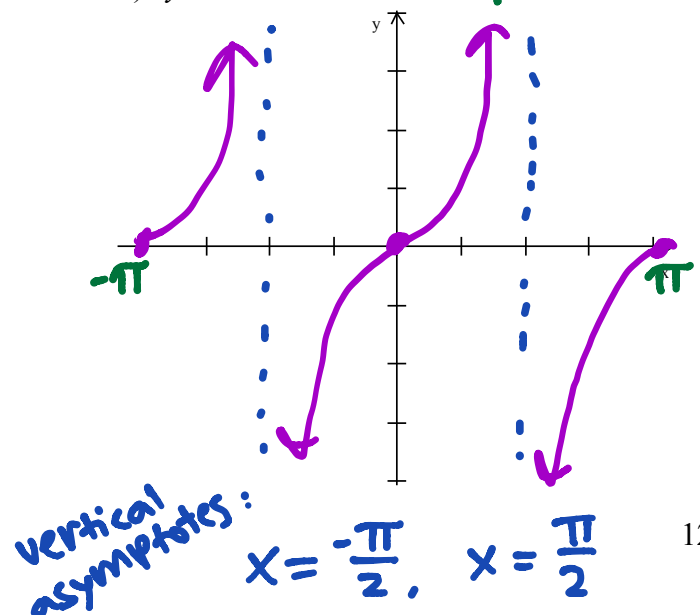
a) $y = -\tan 2x$

Period: $\frac{\pi}{2}$



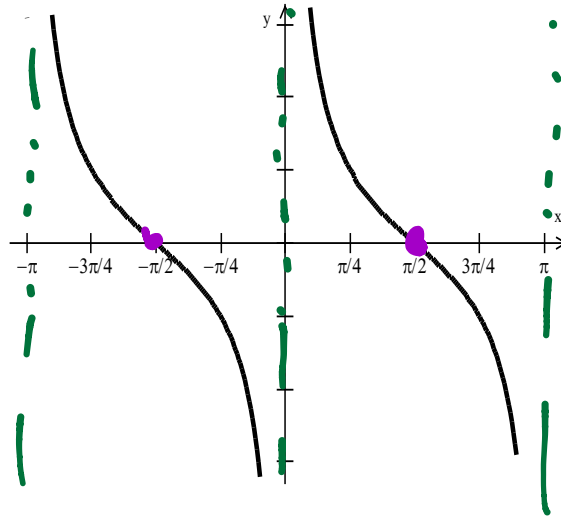
b) $y = 2 \tan x$

Period: π



The Cotangent Function

$$f(x) = \cot x$$



$$\cot x = \frac{\cos x}{\sin x}$$

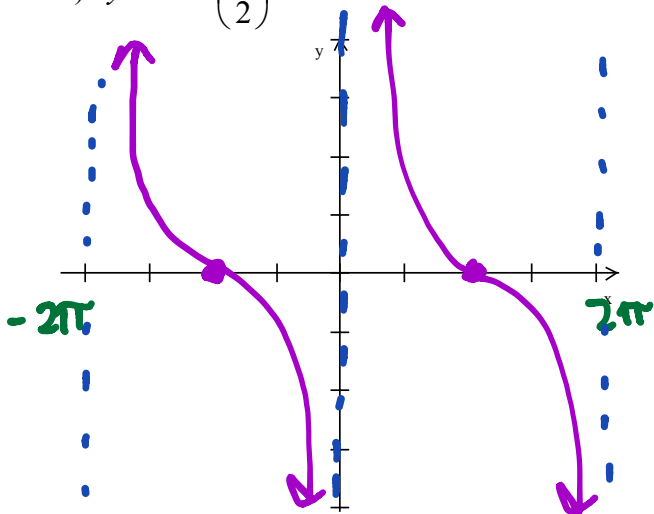
Vertical asymptotes everywhere $\sin x = 0$

Example 1: Graphing Cotangent Functions

Locate the vertical asymptotes and graph two periods of the function. Describe the transformations.

a) $y = 3 \cot\left(\frac{x}{2}\right)$

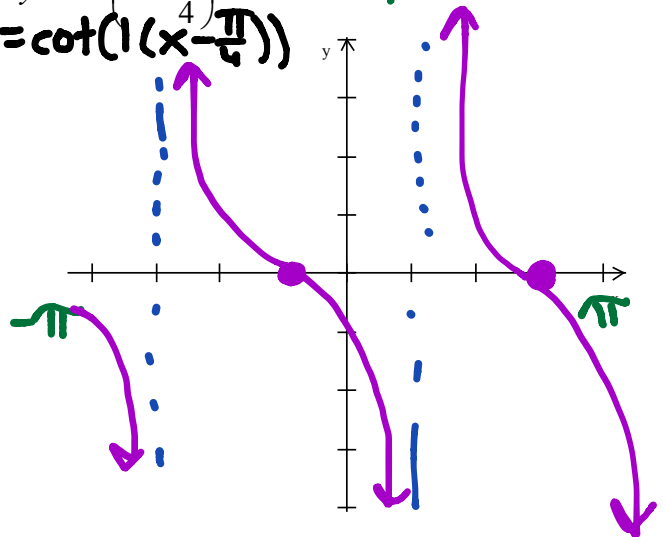
Period: 2π



Vertical asymptotes:
 $x = -2\pi, x = 0, x = 2\pi$

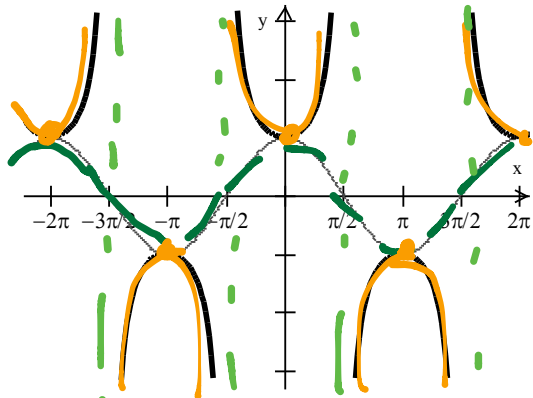
b) $y = \cot\left(x - \frac{\pi}{4}\right)$
 $y = \cot\left(1\left(x - \frac{\pi}{4}\right)\right)$

Period: $\frac{\pi}{1} = \pi$



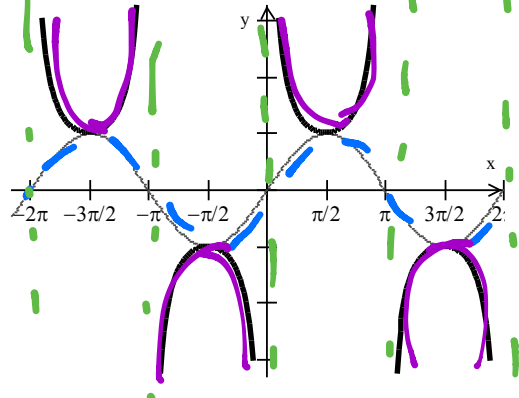
Vertical asymptotes:
 $x = -\frac{3\pi}{4}, x = \frac{\pi}{4}$

$$f(x) = \sec x = \frac{1}{\cos x}$$



Vertical asymptotes everywhere $\cos x = 0$

$$f(x) = \csc x = \frac{1}{\sin x}$$

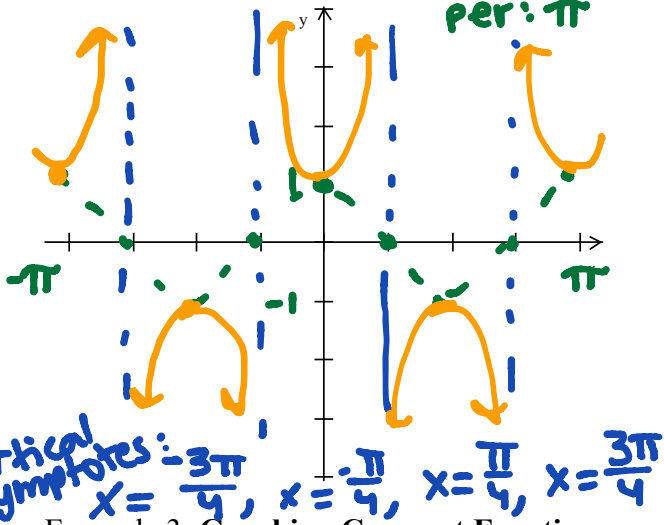


Vertical asymptotes everywhere $\sin x = 0$

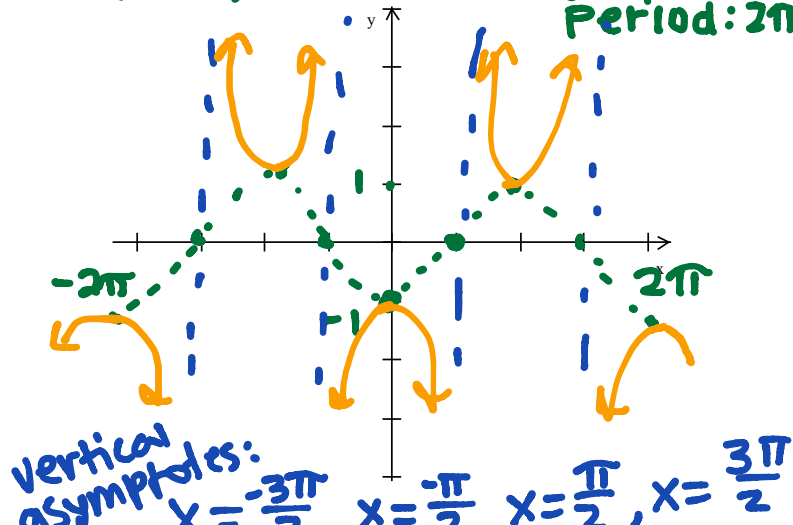
Example 3: Graphing Secant Functions

Locate the vertical asymptotes and graph two periods of the function. Describe the transformations.

a) $y = \sec 2x \Rightarrow \cos 2x$ amp: 1 per: π



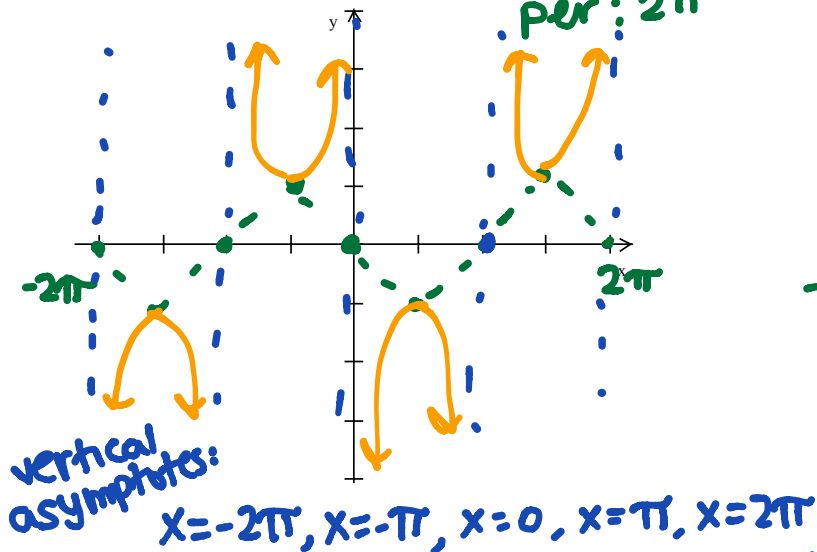
b) $y = \sec x \Rightarrow -\cos x$ amp: 1 Period: 2π



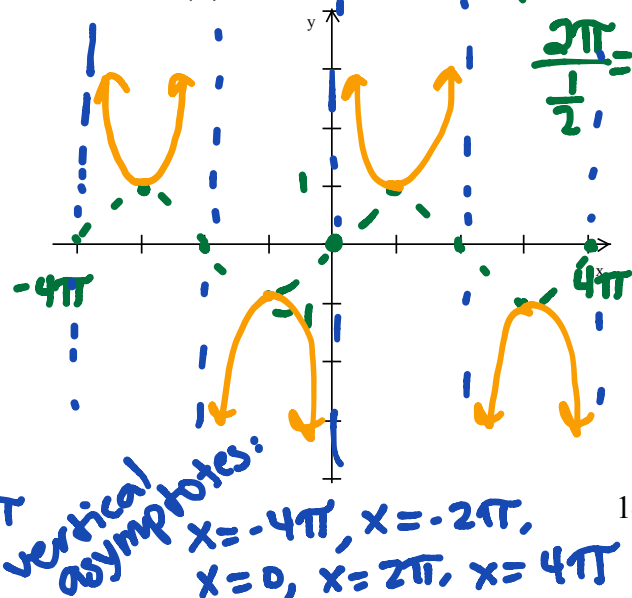
Example 3: Graphing Cosecant Functions

Locate the vertical asymptotes and graph two periods of the function. Describe the transformations.

a) $y = \csc(-x) \Rightarrow \sin(x)$ amp: 1 per: 2π



b) $y = \csc\left(\frac{x}{2}\right) \Rightarrow \sin\left(\frac{x}{2}\right)$ amp: 1 Period: 4π

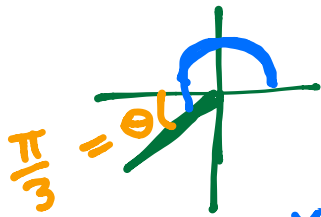


Solving a Trigonometric Equation Algebraically

Example 4: Solve for x in the given interval without using a calculator.

a) $\sec x = -2$

$\cos x = -\frac{1}{2}$ $\pi < x < \frac{3\pi}{2}$ Q3

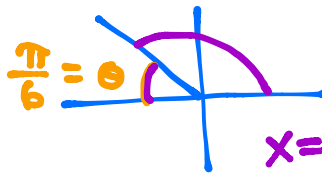


$x = \frac{\pi}{3} + \pi$

$x = \frac{4\pi}{3}$

b) $\cot x = -\sqrt{3}$

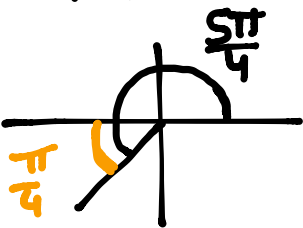
$\tan x = -\frac{1}{\sqrt{3}}$ $\frac{\pi}{2} < x < \pi$ Q2



$x = \pi - \frac{\pi}{6}$

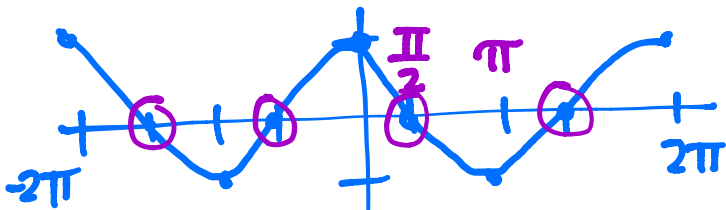
$x = \frac{5\pi}{6}$

$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$



$y = \cos x$ $[-2\pi, 2\pi]$

zeros.



$x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$