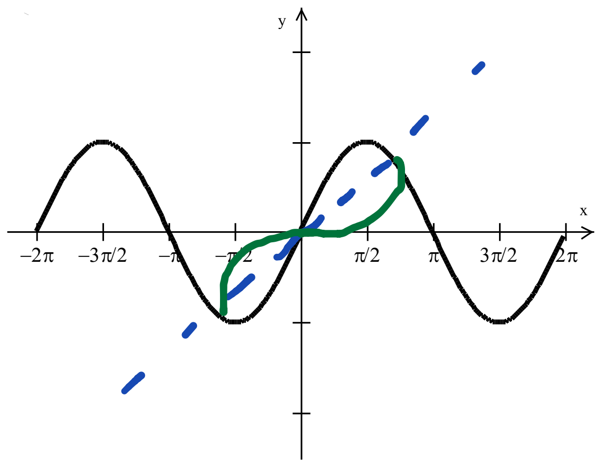
4.7: Inverse Trigonometric Functions

Remember inverse functions are reflections of the graph over the line $y = x$.

$$y = \sin x$$



1. Sketch a graph of $\sin^{-1} x$ on top of the $\sin x$ graph.
2. Is the inverse graph a function?
3. The restrictions on the domain for inverse functions exist in order to make them be functions (pass the vertical line test).

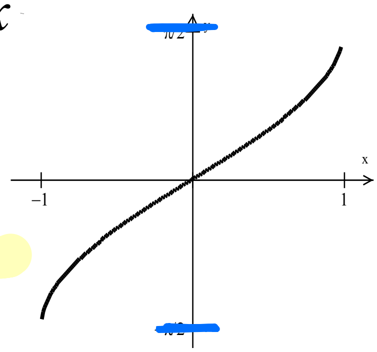
Inverse Sine Function (Arcsine Function)

$$y = \sin^{-1} x$$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q1 Q4



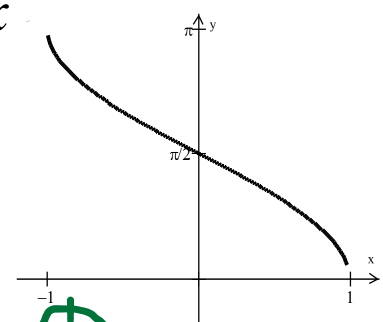
Inverse Cosine Function (Arccosine Function)

$$y = \cos^{-1} x$$

Domain: $[-1, 1]$

Range: $[0, \pi]$

Q1 Q2



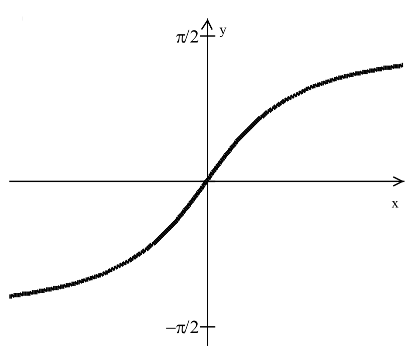
Inverse Tangent Function (Arctangent Function)

$$y = \tan^{-1} x$$

Domain: $(-\infty, \infty)$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q1 Q4



Example 1: **Evaluating Inverse Functions Without a Calculator**

a) $\sin^{-1}\left(\frac{1}{2}\right) = \boxed{30^\circ \text{ or } \pi/6}$

b) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \boxed{135^\circ \text{ or } \frac{3\pi}{4}}$

$\theta = 45^\circ$ $\quad 180 - 45 = 135^\circ$ $\quad \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

c) $\tan^{-1}(1) = \boxed{45^\circ \text{ or } \frac{\pi}{4}}$

d) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-60^\circ \text{ or } \frac{-\pi}{3}}$

$\theta = 60^\circ$

The following equation are always true whenever they are defined:

$$\cancel{\sin}\left(\cancel{\sin}^{-1}(x)\right) = x \qquad \cancel{\cos}\left(\cancel{\cos}^{-1}(x)\right) = x \qquad \cancel{\tan}\left(\cancel{\tan}^{-1}(x)\right) = x$$

On the other hand, the following equations are only true for x values in the "restricted" domain of sin, cos, and tan:

$$\sin^{-1}(\sin(x)) = x \qquad \cos^{-1}(\cos(x)) = x \qquad \tan^{-1}(\tan(x)) = x$$

Example 2: **Evaluating Inverse Functions Without a Calculator**

a) $\sin^{-1}\left(\underbrace{\sin\frac{\pi}{2}}\right)$ $\quad (0,1)$

$\sin^{-1}(1) = \boxed{\frac{\pi}{2}}$

b) $\cancel{\cos}^{-1}\left(\underbrace{\cancel{\cos}\left(\frac{5\pi}{6}\right)}\right)$ $\quad \pi/6$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{\frac{5\pi}{6}}$ $\quad \pi/6$

c) $\cancel{\tan}\left(\underbrace{\cancel{\tan}^{-1}\left(\sqrt{3}\right)}\right) = \sqrt{3}$

d) $\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{2}$

Example 3: **Evaluating Inverse Functions Using a Calculator**

Use a calculator to find the approximate value. Express your answer degrees on example (a) and in radians on example (b).

a) $\sin^{-1}(0.541) = \boxed{32.75^\circ}$

b) $\cos^{-1}(-.875) = \boxed{2.64 \text{ radians}}$