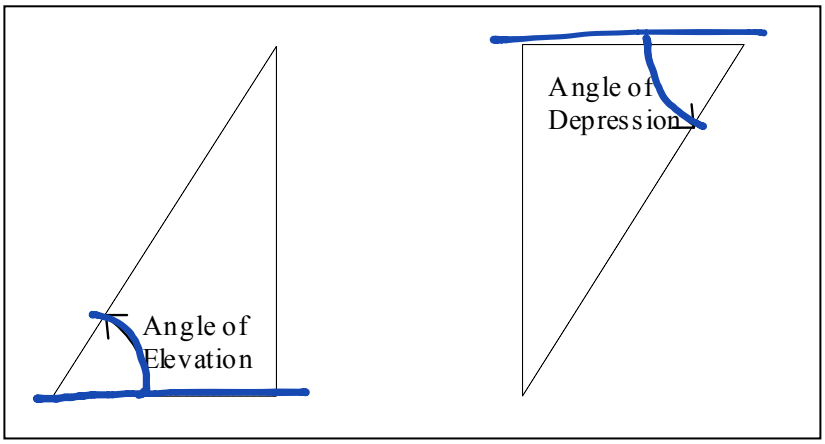
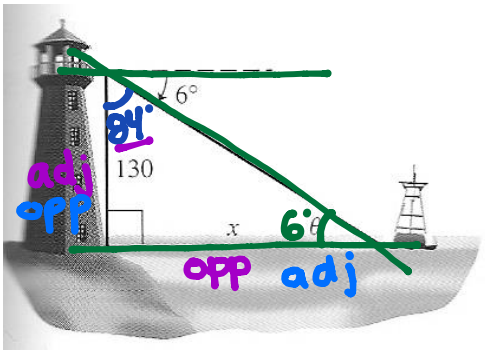
4.8: Solving Problems with Trigonometry



Example 1: **Using Angle of Depression**
The angle of depression of a buoy from the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is 6°. Find the distance x from the base of the lighthouse to the buoy.



$130 \cdot \tan 84 = \frac{x}{130} \cdot 130$

$130\tan 84 = x$

$x \approx 1236.87 \text{ ft}$

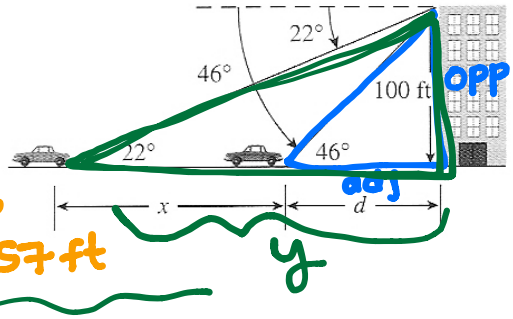
$x \cdot \tan 6 = \frac{130}{x} \cdot x$

$\frac{x\tan 6}{\tan 6} = \frac{130}{\tan 6}$

$x = \frac{130}{\tan 6}$

$x \approx 1236.87 \text{ ft}$

Example 2: **Making Indirect Measurements**
From the top of the 100-ft-tall Altgelt Hall a man observes a car moving toward the building. If the angle of depression of the car changes from 22° to 46° during the period of observation, how far does the car travel?



small triangle:

$d \cdot \tan 46 = \frac{100}{d} \cdot d$

$\frac{d\tan 46}{\tan 46} = \frac{100}{\tan 46}$

$d \approx 96.57 \text{ ft}$

Big Triangle:

$y \cdot \tan 22 = \frac{100}{y} \cdot y$

$\frac{y\tan 22}{\tan 22} = \frac{100}{\tan 22}$

$y \approx 247.5 \text{ ft}$

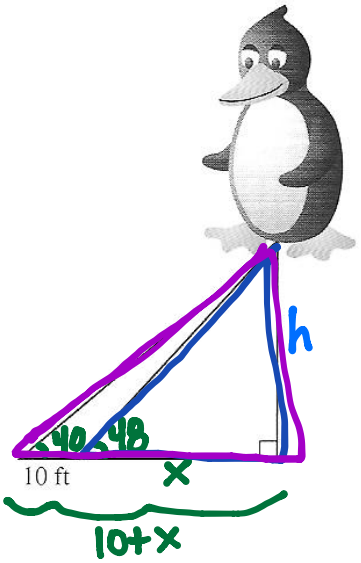
$x = y - d$

$x = 247.5 - 96.57$

$x \approx 150.93 \text{ ft}$

Example 3: **Finding Height Above Ground**

A large, helium-filled penguin is moored at the beginning of a parade route awaiting the start of the parade. Two cables attached to the underside of the penguin make angles of 48° and 40° with the ground and are in the same plane as a perpendicular line from the penguin to the ground. If the cables are attached to the ground 10 feet from each other, how high above the ground is the penguin?



$\tan 48 = \frac{h}{x}$ $\quad \tan 40 = \frac{h}{10+x}$

$h = x\tan 48$ $\quad (x+10)\tan 40 = \frac{x\tan 48}{10+x} \cdot (10+x)$

$(x+10)\tan 40 = x\tan 48$

$x\tan 40 + 10\tan 40 = x\tan 48$

$-x\tan 40 \qquad -x\tan 40$

$10\tan 40 = x\tan 48 - x\tan 40$

$\frac{10\tan 40}{\tan 48 - \tan 40} = \frac{x(\tan 48 - \tan 40)}{\tan 48 - \tan 40}$

$x = \frac{10\tan 40}{\tan 48 - \tan 40} \approx 30.9\text{ft}$

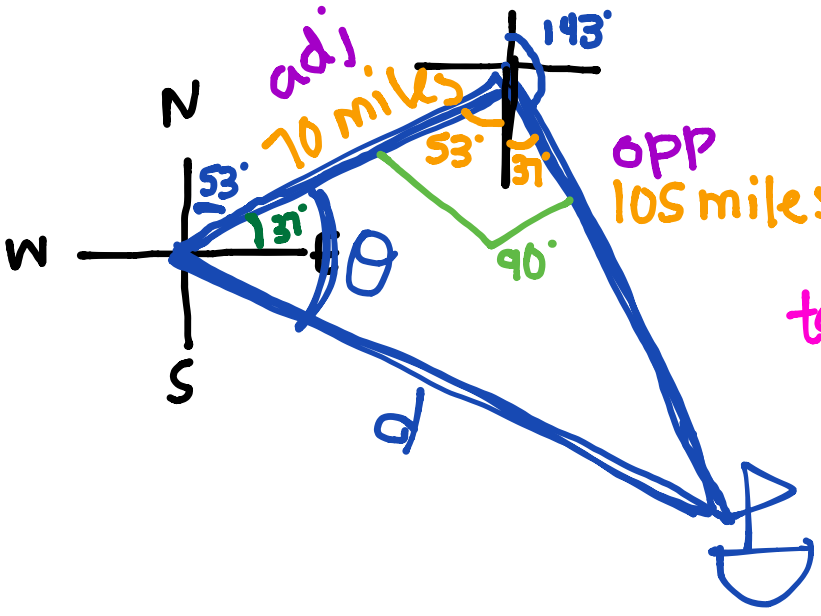
$h = x\tan 48$

$h = (30.9)\tan 48$

$h \approx 34.32\text{ ft}$

Example 4: **Using Trig in Navigation**

A U.S. Coast Guard patrol boat leaves Port Cleveland and averages 35 knots (nautical mph) traveling for 2 hours on a course of 53° and then 3 hours on a course of 143°. What is the boat's bearing and distance from Port Cleveland?



Distance: $a^2 + b^2 = c^2$

$70^2 + 105^2 = d^2$

$d \approx 126.2$ miles

Bearings:

$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{105}{70}\right)$

$\theta = \tan^{-1}\left(\frac{105}{70}\right)$

$\theta = 56.3°$

$53 + 56.3 = 109.3°$