Name ____ Period ___________

Precalculus: Analytic Trigonometry

5.1: Fundamental Identities

Basic Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Odd-Even Identities

Odd Functions	Even Functions	Odd Functions
$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$

Example 1: **Pythagorean Identities**

Convert $\sin^2 x + \cos^2 x = 1$ to $\cot^2 x + 1 = \csc^2 x = \frac{1}{\sin^2 x}$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x \quad ✓$$

Example 2: **Simplifying Factoring and Using Identities**

a) $\sin^3 x + \sin x \cos^2 x$

$\sin x(\underbrace{\sin^2 x + \cos^2 x}_{=1})$

$\sin x(1) = \boxed{\sin x}$

b) $1 - 2\sin x + \sin^2 x$ let $u = \sin x$

$1 - 2u + u^2 = u^2 - 2u + 1$

$(u-1)(u-1) = (u-1)^2$

$\boxed{(\sin x - 1)^2}$

c) $\cos^4 x - 2\cos^2 x + 1$ let $u = \cos^2 x$

$u^2 - 2u + 1 = (u-1)^2$

$= (\underbrace{\cos^2 x - 1}_{\downarrow})^2$

$= (-\sin^2 x)^2 = \boxed{\sin^4 x}$

Example 3: **Simplifying by Expanding and Using Identities**

Simplify the expression $\dfrac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$

$= \dfrac{\sec^2 x - \sec x + \sec x - 1}{\sin^2 x} = \dfrac{\sec^2 x - 1}{\sin^2 x}$ $=\tan^2 x$

$\dfrac{\tan^2 x}{\sin^2 x} = \tan^2 x\left(\dfrac{1}{\sin^2 x}\right) = \left(\dfrac{\cancel{\sin^2 x}}{\cos^2 x}\right)\left(\dfrac{1}{\cancel{\sin^2 x}}\right)$

$= \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

Example 4: **Simplifying by Combining Fractions and Using Identities**

Simplify the expression $\dfrac{(\cos x)}{(\cos x)}\dfrac{\cos x}{1 - \sin x} - \dfrac{\sin x}{\cos x}\dfrac{(1-\sin x)}{(1-\sin x)}$ LCD: $(1-\sin x)(\cos x)$

$\dfrac{\cos^2 x}{\cos x(1-\sin x)} - \dfrac{\sin x(1-\sin x)}{\cos x(1-\sin x)} = \dfrac{\cos^2 x - (\sin x - \sin^2 x)}{\cos x(1-\sin x)}$

$\dfrac{\underbrace{\cos^2 x - \sin x + \sin^2 x}_{=1}}{\cos x(1-\sin x)} = \dfrac{\cancel{1-\sin x}}{\cos x\,\cancel{(1-\sin x)}} = \dfrac{1}{\cos x}$

$= \boxed{\sec x}$

Example 5: Simplify each expression.

a) $\cos x \underbrace{\sec(-x)}$ $\sec(-x) = \sec x$

$\cos x \sec x$

$\cancel{\cos x}\left(\frac{1}{\cancel{\cos x}}\right) = \boxed{1}$

b) $\sec^2(-x) - \tan^2(-x)$

$\sec^2(-x) = (\sec(-x))^2$

$= (\sec(x))^2 = \sec^2 x$

$\tan^2(-x) = (\tan(-x))^2$

$= (-\tan(x))^2 = \tan^2 x$

$\tan^2 x + 1 = \sec^2 x$

$-\tan^2 x \qquad -\tan^2 x$

$1 = \underline{\sec^2 x - \tan^2 x}$

$\underline{\sec^2 x - \tan^2 x}$

$= \boxed{1}$

Example 6: **Solving a Trigonometric Equation**

Find all values of x in the interval $[0, 2\pi)$ that solve the equation.

a) $2\cos x + \sqrt{3} = 0$

b) $\sin^2 x - 2\sin x = 0$

c) $\dfrac{\cos^3 x}{\sin x} = \cot x$

Example 7: **Solving a Trigonometric Equation by Factoring**

Find all solutions to the trigonometric equation

a) $2\sin^2 x + \sin x = 1$

1. $\tan x = \dfrac{\sin x}{\cos x}$ $\qquad \tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$

2. $\dfrac{1}{\csc x} = \sin x \qquad \dfrac{1}{\cos x} = \sec x \qquad \dfrac{\cos x}{\sin x} = \cot x$

3. $\sin^2 x + \cos^2 x = 1$

$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

$$\underbrace{\cos^2 x - 1} = -\sin^2 x$$

$$\begin{array}{rcl}
\sin^2 x + \cos^2 x & = & 1 \\
-1 & & -1 \\
\hline
\sin^2 x + \underbrace{\cos^2 x - 1} & = & 0 \\
-\sin^2 x \qquad\qquad & & -\sin^2 x \\
\hline
\cos^2 x - 1 & = & -\sin^2 x
\end{array}$$