Example 5: Simplify each expression.

a) $\cos x \sec(-x)$

b) $\sec^2(-x) - \tan^2(-x)$

Example 6: **Solving a Trigonometric Equation**

Find all values of x in the interval $[0, 2\pi)$ that solve the equation.

a) $2\cos x + \sqrt{3} = 0$

$-\sqrt{3} \quad -\sqrt{3}$

$\frac{2\cos x}{2} = \frac{-\sqrt{3}}{2}$

$\cos x = \frac{-\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

b) $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad$ $\sin x - 2 = 0$, $\sin x \neq 2$

$x = 0, \pi$

c) $\frac{\cos^3 x}{\sin x} = \cot x$

$\frac{\cos^3 x}{\sin x} = \frac{\cos x}{\sin x}$

$\cos^3 x = \cos x$

$-\cos x \quad -\cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ $\quad$ $\cos^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Example 7: **Solving a Trigonometric Equation by Factoring**

Find all solutions to the trigonometric equation

a) $2\sin^2 x + \sin x = 1$

$-1 \quad -1$

$2\sin^2 x + \sin x - 1 = 0$

$u = \sin x$

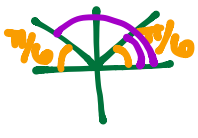
$2u^2 + u - 1 = 0$

$(2u - 1)(u + 1) = 0$

$2u - 1 = 0$ $\quad$ $u + 1 = 0$

$u = \frac{1}{2}$ $\quad$ $u = -1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{3\pi}{2}$

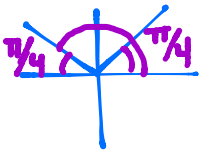
$$x = \begin{cases} \frac{\pi}{6} + 2\pi k \\ \frac{5\pi}{6} + 2\pi k \\ \frac{3\pi}{2} + 2\pi k \end{cases}$$ for all integers k

period

$\sin 2x = \frac{1}{\sqrt{2}}$ $[0, 2\pi)$ period: π

$u = 2x$

$\sin u = \frac{1}{\sqrt{2}}$



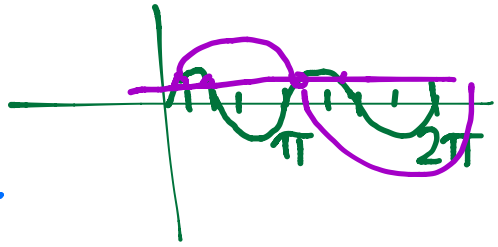
$u = \frac{\pi}{4}, \frac{3\pi}{4}$

$\frac{2x}{2} = \frac{\pi}{4} \cdot \frac{1}{2}$ $\frac{2x}{2} = \frac{3\pi}{4} \cdot \frac{1}{2}$

$x = \frac{\pi}{8}$ $x = \frac{3\pi}{8}$

$+\pi$ $+\pi$

$x = \frac{9\pi}{8}$ $x = \frac{11\pi}{8}$



$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

$\cos^2 x + \cos x = \sin^2 x$ $0 \le x < 360^\circ$

$\cos^2 x + \cos x - \sin^2 x = 0$

$\cos^2 x + \cos x - (1 - \cos^2 x) = 0$

$\cos^2 x + \cos x - 1 + \cos^2 x = 0$

$2\cos^2 x + \cos x - 1 = 0$

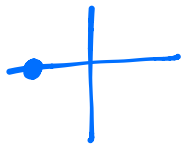
$u = \cos x$

$2u^2 + u - 1 = 0$

$(2u - 1)(u + 1) = 0$

$u = \frac{1}{2}$ $u = -1$

$\cos x = \frac{1}{2}$ $\cos x = -1$



$x = 60^\circ, 300^\circ$ $x = 180^\circ$

$x = 60^\circ, 180^\circ, 300^\circ$