

Precalculus: 5.1-5.4 Review

Write all of the identities (These must be memorized).

1. Pythagorean Identities (all 3):

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \\ 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

2. Sum and Difference Identities:

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \sin \beta \cos \alpha \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \end{aligned}$$

3. Double Angle Identities:

$$\begin{aligned} \sin(2u) &= 2 \sin u \cos u \\ \cos(2u) &= \frac{2 \tan u}{1 + \tan^2 u} \\ \tan(2u) &= \frac{\cos^2 u - \sin^2 u}{2 \cos^2 u - 1} \\ &= \frac{1 - 2 \sin^2 u}{1 - 2 \sin^2 u} \end{aligned}$$

Simplify each expression.

4. $\cot x \sin x \sec x$

$$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} \cdot \frac{1}{\cos x} = \boxed{1}$$

9. $\frac{\sin(-x) \cot(-x)}{\text{odd odd}}$

$$\begin{aligned} &(-\sin x)(-\cot x) \\ &(-\sin x) \left(\frac{-\cos x}{\sin x} \right) = \boxed{\cos x} \end{aligned}$$

5. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$

$$\begin{aligned} \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}} \\ \sin^2 x + \cos^2 x = \boxed{1} \end{aligned}$$

10. $\frac{-\tan^2 x - 1}{\sec^2 x} = \frac{-(\tan^2 x + 1)}{\sec^2 x}$

$$= \frac{-\sec^2 x}{\sec^2 x} = \boxed{-1}$$

6. $\cos^2 x - 1$

$$\boxed{-\sin^2 x}$$

11. $1 - \frac{1}{\cos^2 x}$

$$1 - \sec^2 x = \boxed{-\tan^2 x}$$

7. $1 - \sec^2 x$

$$\boxed{-\tan^2 x}$$

12. $\sin\left(\frac{\pi}{2} - x\right)$

$$\begin{aligned} \sin \frac{\pi}{2} \cos x - \sin x \cos \frac{\pi}{2} \\ 1 \cdot \cos x - \sin x \cdot 0 \\ \cos x - 0 = \boxed{\cos x} \end{aligned}$$

8. $\cos x \sin^2 x - \cos x$

$$\begin{aligned} \cos x (\sin^2 x - 1) \\ \cos x (-\cos^2 x) \\ \boxed{-\cos^3 x} \end{aligned}$$

13. $\cos\left(x - \frac{3\pi}{2}\right)$

$$\begin{aligned} \cos x \cos \frac{3\pi}{2} + \sin x \sin \frac{3\pi}{2} \\ \cos x \cdot 0 + \sin x (-1) \\ \boxed{-\sin x} \end{aligned}$$

14. $\sin x \cos 3x + \sin 3x \cos x$

$\sin(x+3x) = \sin 4x$

15. $\cos 35^\circ \cos 25^\circ - \sin 35^\circ \sin 25^\circ$

$\cos(35+25) = \cos 60$
 $= \frac{1}{2}$

16. $2 \sin x \cos x$

$\sin 2x$

17. $\cos^2 x - \sin^2 x$

$\cos 2x$

Solve each equation for $0 \leq \theta < 2\pi$

18. $\sin \theta = \frac{1}{2}$ $+$

$\theta = 30^\circ$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

19. $\cos \theta = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

20. $\sin \theta - 1 = 0$

$\sin \theta = 1$

$\theta = \frac{\pi}{2}$

21. $2 - 3 \cos \theta = 2$

$-3 \cos \theta = 0$
 $\cos \theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

* 22. $\sqrt{3} \cot \theta = -1$

$\cot \theta = -\frac{1}{\sqrt{3}}$

$\tan \theta = -\frac{\sqrt{3}}{1}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

23. $\sqrt{\cos^2 \theta} = \frac{1}{2}$

$\cos \theta = \pm \sqrt{\frac{1}{2}}$

$\cos \theta = \pm \frac{1}{\sqrt{2}}$

$+$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

24. $\cos 2\theta = -\frac{\sqrt{3}}{2}$

$u = 2\theta$

$\cos u = -\frac{\sqrt{3}}{2}$

$u = \frac{5\pi}{6}, u = \frac{7\pi}{6}$

$2\theta = \frac{5\pi}{6}, 2\theta = \frac{7\pi}{6}$

$\theta = \frac{5\pi}{12}, \theta = \frac{7\pi}{12}$

$\frac{17\pi}{12}$

$\frac{19\pi}{12}$

25. $\sin\left(\frac{\theta}{2}\right) = 0$

$u = \frac{\theta}{2}$

$\sin u = 0$

$u = 0, \pi$

$\frac{\theta}{2} = 0$

$\frac{\theta}{2} = \pi$

$\theta = 0$

$\theta = 2\pi$

26. $\tan 2\theta = \frac{\sqrt{3}}{3}$

$u = 2\theta$

$\tan u = \frac{\sqrt{3}}{3}$

$u = \frac{\pi}{6}$

$2\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{25\pi}{12}$

27. $\sin 2\theta = \cos \theta$

$2 \sin \theta \cos \theta = \cos \theta$

$2 \sin \theta \cos \theta - \cos \theta = 0$

$\cos \theta (2 \sin \theta - 1) = 0$

$\cos \theta = 0$ $2 \sin \theta - 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin \theta = \frac{1}{2}$
 $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

bigger than 2π

Find all solutions to each equation in radians.

28. $\cos\theta + 5 = -2$

$\cos\theta = -7$

NO solution

29. $\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\theta = \left\{ \begin{array}{l} \frac{2\pi}{3} + 2\pi k \\ \frac{4\pi}{3} + 2\pi k \end{array} \right.$

30. $\tan\theta = -1$

$\theta = \frac{\pi}{4} + \pi k$

31. $2\sin 2\theta - 1 = 0$

$\frac{2\sin 2\theta}{2} = \frac{1}{2}$

$\sin 2\theta = \frac{1}{2}$

$u = 2\theta$

$\sin u = \frac{1}{2}$

$u = \frac{\pi}{6} \quad u = \frac{5\pi}{6}$

$2\theta = \frac{\pi}{6} \quad 2\theta = \frac{5\pi}{6}$

$\theta = \frac{\pi}{12} \quad \theta = \frac{5\pi}{12}$

32. $\frac{4\cos 3\theta}{4} = \frac{-2}{4}$

$\cos 3\theta = -\frac{1}{2}$

$u = 3\theta$

$\cos u = -\frac{1}{2}$

$u = \frac{2\pi}{3} \quad u = \frac{4\pi}{3}$

$3\theta = \frac{2\pi}{3} \quad 3\theta = \frac{4\pi}{3}$

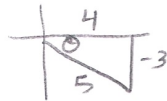
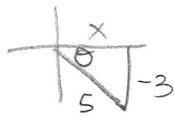
$\theta = \frac{2\pi}{9} \quad \theta = \frac{4\pi}{9}$

$\theta = \left\{ \begin{array}{l} \frac{2\pi}{9} + \frac{2\pi}{3}k \\ \frac{4\pi}{9} + \frac{2\pi}{3}k \end{array} \right.$

Use the given information to find the exact value of the expression.

33. Given: $\sin\theta = -\frac{3}{5}$, θ lies in quadrant IV. Find $\sin 2\theta$ and $\cos 2\theta$.

3 opposite
5 hyp



$\sin\theta = -\frac{3}{5}$
 $\cos\theta = \frac{4}{5}$

$(-3)^2 + x^2 = 5^2$
 $x = 4$

$\sin 2\theta = 2\sin\theta\cos\theta$ | $\cos 2\theta = 1 - 2\sin^2\theta$
 $= 2(-\frac{3}{5})(\frac{4}{5})$ | $1 - 2(-\frac{3}{5})^2$

$\sin 2\theta = -\frac{24}{25}$

$1 - 2(\frac{9}{25})$

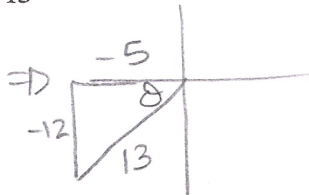
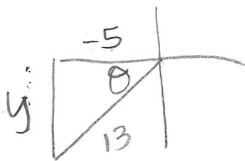
$1 - \frac{18}{25}$

$\frac{25 - 18}{25} = \frac{7}{25}$

$\cos 2\theta = \frac{7}{25}$

you can choose any of the 3 identities

34. Given: $\cos\theta = -\frac{5}{13}$, θ lies in quadrant III. Find $\sin 2\theta$ and $\cos 2\theta$.



$(-5)^2 + y^2 = 13^2$

$y = 12$

$\sin\theta = -\frac{12}{13}$

$\cos\theta = -\frac{5}{13}$

$\sin 2\theta = 2\sin\theta\cos\theta$
 $= 2(-\frac{12}{13})(-\frac{5}{13})$

$\sin 2\theta = \frac{120}{169}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
 $(-\frac{5}{13})^2 - (-\frac{12}{13})^2$
 $\frac{25}{169} - \frac{144}{169}$

$\cos 2\theta = -\frac{119}{169}$