5.3: Sums and Difference Identities

Sum and Difference Identities

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Example 1: **Using Sine-of-a-Difference Identity**
Find the exact value of $\sin 15°$ without using a calculator.

$15° = 45° - 30°$

$\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30 - \sin 30 \cos 45°$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{3}-1}{2\sqrt{2}}$

Example 2: **Confirming Cofunction Identities**

Prove the identities (a) $\cos\left(\frac{\pi}{2}-x\right)=\sin x$ and (b) $\sin\left(\frac{\pi}{2}-x\right)=\cos x$

(a)

$\cos\left(\frac{\pi}{2}-x\right)$

$=\cos\frac{\pi}{2}\cos x+\sin\frac{\pi}{2}\sin x$

$= 0\cdot\cancel{\cos x} + (1)\sin x$

$= \sin x$ ✓

(0,1)

(b)

$\sin\left(\frac{\pi}{2}-x\right)$

$= \sin\frac{\pi}{2}\cos x - \sin x\cos\frac{\pi}{2}$

$= (1)\cos x - \cancel{\sin x(0)}$

$= \cos x$ ✓

Example 3: **Using the Sum/Difference Formulas**
Write each of the following expressions as the sine or cosine of an angle.

a) $\sin 22^\circ \cos 13^\circ \pm \cos 22^\circ \sin 13^\circ$

$\sin(22+13)$

$= \boxed{\sin(35^\circ)}$

b) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} \pm \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= \cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right) = \boxed{\cos\left(\frac{\pi}{12}\right)}$

c) $\sin x \sin 2x - \cos x \cos 2x$

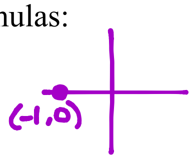
$-\cos x \cos 2x + \sin x \sin 2x$

$= -1(\cos x \cos 2x - \sin x \sin 2x)$

$= -1(\cos(x+2x)) = \boxed{-\cos(3x)}$

Example 4: **Proving Reduction Formulas**
Prove the reduction formulas:

a) $\sin(x+\pi) = -\sin x$



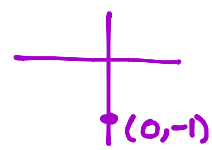
$\sin(x+\pi) =$

$= \sin x \cos\pi + \sin\pi \cos x$

$= \sin x(-1) + \cancel{(0)\cos x}$

$= -\sin x$ ✓

b) $\cos\left(x+\frac{3\pi}{2}\right) = \sin x$



$\cos\left(x+\frac{3\pi}{2}\right)$

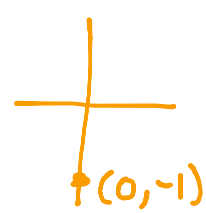
$= \cos x \cos\frac{3\pi}{2} - \sin x \sin\frac{3\pi}{2}$

$= \cancel{\cos x(0)} - \sin x(-1)$

$= \sin x$ ✓

Example 5: **Proving Tangent Reduction Formulas**

Prove the reduction formula: $\tan\left(\theta - \frac{3\pi}{2}\right) = -\cot\theta$



$$\tan\left(\theta-\frac{3\pi}{2}\right) = \frac{\sin\left(\theta-\frac{3\pi}{2}\right)}{\cos\left(\theta-\frac{3\pi}{2}\right)} = \frac{\sin\theta\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2}\cos\theta}{\cos\theta\cos\frac{3\pi}{2} + \sin\theta\sin\frac{3\pi}{2}}$$

$$= \frac{\cancel{\sin\theta(0)} - (-1)\cos\theta}{\cancel{\cos\theta(0)} + \sin\theta(-1)} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta \checkmark$$

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QR: 1-4
EX: 1-7, 11, 13, 15, 16, 19, 23, 25, 27, 37, 47, 48