5.4: Multiple-Angle Identities

MEMORIZED

Double Angle Identities

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$\cos 2u = 1 - 2\sin^2 u$

$\cos 2u = 2\cos^2 u - 1$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Half-Angle Identities

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

$\tan\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha}$

$\tan\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{\sin\alpha}$

Power-Reducing Identities

$\sin^2 u = \dfrac{1-\cos 2u}{2}$

$\cos^2 u = \dfrac{1+\cos 2u}{2}$

$\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

Example 1: **Proving a Double-Angle Identity**

a) Prove the identity: $\sin 2u = 2\sin u \cos u$

$\sin 2u = \sin(u+u)$

$= \sin u\cos u + \sin u\cos u$

$= 2\sin u\cos u$ ✓

b) Prove the identities for $\cos 2u$

$\cos 2u = \cos(u+u)$

$= \cos u\cos u - \sin u\sin u$

$= \cos^2 u - \sin^2 u$ ✓

$\cos^2 u - \sin^2 u$

$= (1-\sin^2 u) - \sin^2 u$

$= 1 - 2\sin^2 u$ ✓

$\cos^2 u - \sin^2 u$

$= \cos^2 u - (1-\cos^2 u)$

$= \cos^2 u - 1 + \cos^2 u$

$= 2\cos^2 u - 1$ ✓

c) Prove the identity for $\tan 2u$

$\tan 2u = \tan(u+u)$

$= \dfrac{\tan u + \tan u}{1 - \tan u\tan u} = \dfrac{2\tan u}{1-\tan^2 u}$ ✓

Example 2: **Proving an Identity**
Prove the identity: $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (\cos^2\theta - \sin^2\theta)(1)$

$= \cos^2\theta - \sin^2\theta = \cos 2\theta$ ✓

Example 4: **Using a Double-Angle Identity**
Solve algebraically in the interval $[0, 2\pi)$:

a) $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$
$-\cos x \quad -\cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) $2\sin 2x - 1 = 0$ period: π

$u = 2x$

$2\sin u - 1 = 0$

$\sin u = \frac{1}{2}$

$u = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

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QR 1-3, 6, 8 EX 1-7, 15, 20, 23, 25