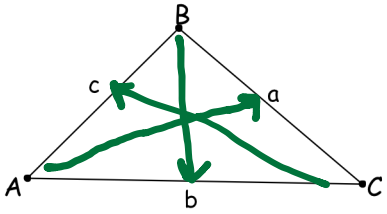
5.5 pg 484:
QR 7,9
EX 1,2,5,9,13-18,19,20,27-32,38,40,51

5.5: The Law of Sines

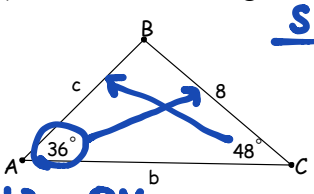
Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1: Solving a Triangle Given Two Angles and a Side

a) Solve the triangle.



$\frac{\sin 36°}{8} = \frac{\sin 48°}{c}$

$c\sin 36 = 8\sin 48$

$\frac{c\sin 36}{\sin 36} = \frac{8\sin 48}{\sin 36}$

$c = \frac{8\sin 48}{\sin 36}$

$c = 10.11$

$\frac{\sin 36}{8} = \frac{\sin 96}{b}$

$b\sin 36 = 8\sin 96$

$\frac{b\sin 36}{\sin 36} = \frac{8\sin 96}{\sin 36}$

$b = \frac{8\sin 96}{\sin 36}$

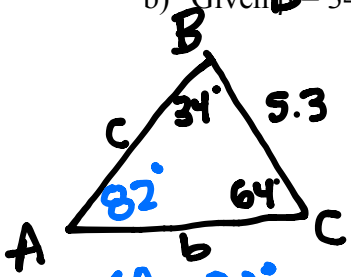
$b = 13.54$

$36 + 48 = 84$

$\angle B = 180 - 84$

$\angle B = 96°$

b) Given $B = 34°$, $C = 64°$, and $a = 5.3$, solve the triangle.



$\angle A = 82°$

$\frac{\sin 82°}{5.3} = \frac{\sin 34°}{b}$

$b\sin 82 = 5.3\sin 34$

$\frac{b\sin 82}{\sin 82} = \frac{5.3\sin 34}{\sin 82}$

$b = \frac{5.3\sin 34}{\sin 82}$

$b = 2.99$

$\frac{\sin 82}{5.3} = \frac{\sin 64}{c}$

$c\sin 82 = 5.3\sin 64$

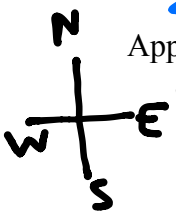
$\frac{c\sin 82}{\sin 82} = \frac{5.3\sin 64}{\sin 82}$

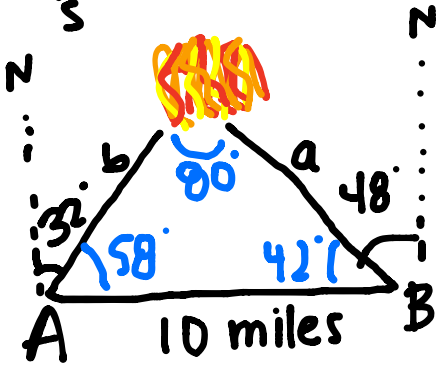
$c = \frac{5.3\sin 64}{\sin 82}$

$c = 4.81$

Applications:

a) Forest Ranger Chris Johnson at ranger station A sights a fire in the direction 32° east of north. Ranger Rick Thorpe at ranger station B , 10 miles due east of A, sights the same fire on a line 48° west of north. Find the distance from each ranger station to the fire.





$\frac{\sin 80}{10} = \frac{\sin 42}{b}$

$b\sin 80 = 10\sin 42$

$\frac{b\sin 80}{\sin 80} = \frac{10\sin 42}{\sin 80}$

$b = \frac{10\sin 42}{\sin 80}$

$b = 6.79$ miles

$\frac{\sin 80}{10} = \frac{\sin 58}{a}$

$a\sin 80 = 10\sin 58$

$\frac{a\sin 80}{\sin 80} = \frac{10\sin 58}{\sin 80}$

$a = \frac{10\sin 58}{\sin 80}$

$a = 8.61$ miles

The Ambiguous Case (SSA)

For two sides and a nonincluded angle, there are several possibilities. To decide on which case you are working with:

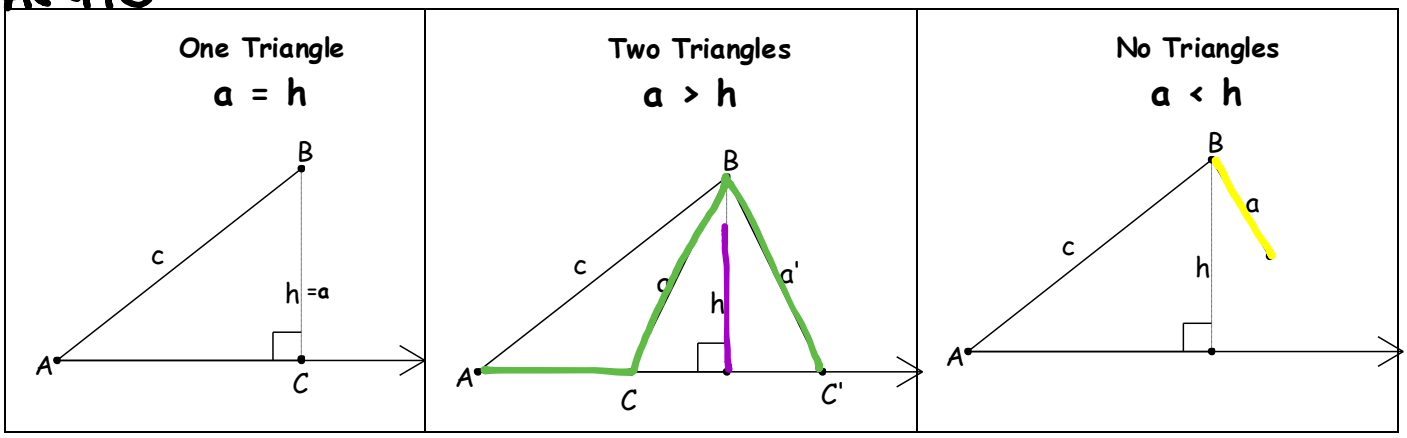
1st: Draw an angle of approximate size α in standard position with terminal side of length *c*, don't draw side a yet.

2nd: Let h be the distance from B to the initial side of α.

3rd: Find h.

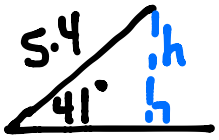
4th: Compare h to the side a to determine if there is one triangle, two triangles or no triangles formed.

ACUTE



Example 2: Determine the number of triangles with the given parts and solve each triangle.

a) Given $A = 41^\circ$, $a = 3.3$, and $b = 5.4$.



$\sin 41 = \frac{h}{5.4}$

$h = 5.4 \sin 41$

$h = 3.54$

$a < h$

$3.3 < 3.54$

NO TRIANGLES

b) Given $B = 56.3^\circ$, $a = 8.3$, and $b = 7.6$.

$\sin 56.3 = \frac{h}{8.3}$

$h = 8.3 \sin 56.3$

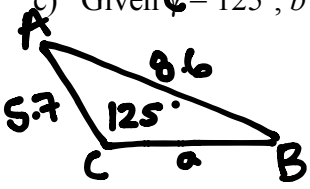
$h = 6.9$

$b > h$

$7.6 > 6.9$

TWO TRIANGLES

c) Given $C = 125^\circ$, $b = 5.7$, and $c = 8.6$.



$\frac{\sin 125}{8.6} = \frac{\sin B}{5.7}$

$5.7 \sin 125 = 8.6 \sin B$

$\frac{5.7 \sin 125}{8.6} = \frac{8.6 \sin B}{8.6}$

$\sin B = \frac{5.7 \sin 125}{8.6}$

$\angle B = 32.88$

$\angle A = 180 - 125 - 32.9$

$\angle A = 22.1^\circ$

$\frac{\sin 125}{8.6} = \frac{\sin 22.1}{a}$

$a \sin 125 = 8.6 \sin 22.1$

$\frac{a \sin 125}{\sin 125} = \frac{8.6 \sin 22.1}{\sin 125}$

$a = \frac{8.6 \sin 22.1}{\sin 125}$

$a = 3.95$

d) Given $B = 22^\circ$, $c = 4.9$, and $b = 2.5$.

$\sin 22 = \frac{h}{4.9}$

$h = 4.9 \sin 22$

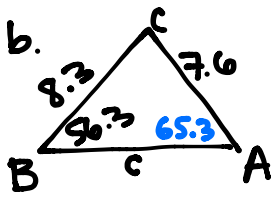
$h = 1.84$

$b > h$

$2.5 > 1.84$

TWO TRIANGLES

b.



$\frac{\sin 56.3}{7.6} = \frac{\sin A}{8.3}$

$\frac{8.3\sin 56.3}{7.6} = \frac{7.6\sin A}{7.6}$

$\sin A = \frac{8.3\sin 56.3}{7.6}$

$A = \sin^{-1}\left(\frac{8.3\sin 56.3}{7.6}\right)$

$\angle A = 65.31°$

$\angle C = 180 - 56.3 - 65.3$

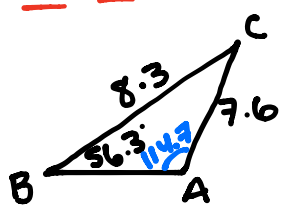
$\angle C = 58.4°$

$\frac{\sin 56.3}{7.6} = \frac{\sin 58.4}{c}$

$\frac{c\sin 56.3}{\sin 56.3} = \frac{7.6\sin 58.4}{\sin 56.3}$

$c = \frac{7.6\sin 58.4}{\sin 56.3}$

$c = 7.8$



$\angle A = 180 - 65.3$

$\angle A = 114.7°$

$\angle C = 180 - 56.3 - 114.7$

$\angle C = 9°$

$\frac{\sin 56.3}{7.6} = \frac{\sin 9}{c}$

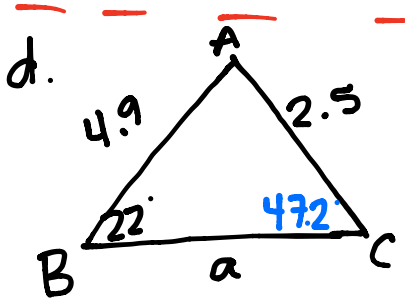
$\frac{c\sin 56.3}{\sin 56.3} = \frac{7.6\sin 9}{\sin 56.3}$

$c = \frac{7.6\sin 9}{\sin 56.3}$

$c = 1.43$

---

d.



$\frac{\sin 22}{2.5} = \frac{\sin C}{4.9}$

$\frac{4.9\sin 22}{2.5} = \frac{2.5\sin C}{2.5}$

$\sin C = \frac{4.9\sin 22}{2.5}$

$\angle C = 47.2°$

$\angle A = 180 - 22 - 47.2$

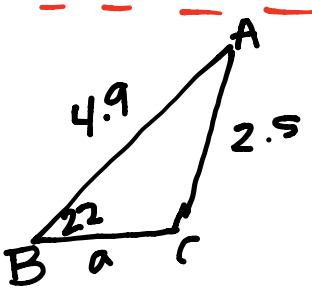
$\angle A = 110.8°$

$\frac{\sin 22}{2.5} = \frac{\sin 110.8}{a}$

$\frac{a\sin 22}{\sin 22} = \frac{2.5\sin 110.8}{\sin 22}$

$a = \frac{2.5\sin 110.8}{\sin 22}$

$a = 6.24$



$\angle C = 180 - 47.2$

$\angle C = 132.8°$

$\angle A = 180 - 22 - 132.8$

$\angle A = 25.2°$

$\frac{\sin 22}{2.5} = \frac{\sin 25.2}{a}$

$a = 2.84$