

Precalculus: Chapter 5 Review

Determine the exact value of each expression.

1. $\sin\left(\frac{\pi}{2}\right) = 1$

2. $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

3. $\tan\left(\frac{-7\pi}{4}\right) = 1$

4. $\cot(\pi) = \text{undefined}$

5. $\sec\left(\frac{5\pi}{6}\right) = \frac{-2}{\sqrt{3}}$

6. $\csc\left(\frac{3\pi}{2}\right) = -1$

Simplify each expression.

7. $(1 - \sin \alpha)(1 + \sin \alpha)$

$\cos^2 \alpha$

8. $(\cos \alpha - \sin \alpha)^2 + (\cos \alpha + \sin \alpha)^2$

2

9. $\frac{1 - \cos 2x}{\sin x}$

$2 \sin x$

10. $\frac{\cos^2 x - \sin^2 x}{\sin 2x}$

$\cot(2x)$

11. $\sec x \cot x \sin 2x$

$2 \cos x$

12. $\frac{1}{1 + \sin x} - \frac{\sin(-x)}{\cos^2 x}$

$\sec^2 x$

13. $\frac{1}{1 - \cos y} + \frac{1}{1 + \cos y}$

$2 \csc^2 y$

14. $\frac{\tan\left(\frac{\pi}{5}\right) + \tan\left(\frac{\pi}{10}\right)}{1 - \tan\left(\frac{\pi}{5}\right)\tan\left(\frac{\pi}{10}\right)}$

$\tan\left(\frac{3\pi}{10}\right)$

15. $\sin 3\theta \cos 6\theta - \cos 3\theta \sin 6\theta$

$-\sin(3\theta)$

16. $1 - \frac{1}{\cos^2 x}$

$-\tan^2 x$

Use identities to find the exact values of the remaining five trigonometric function at x .

$$17. \cos x = \frac{-5}{13} \text{ and } \frac{\pi}{2} < x < \pi$$

$$\sin x = \frac{12}{13} \quad \csc x = \frac{13}{12} \quad \cot x = \frac{-5}{12}$$

$$\tan x = \frac{-12}{-5} \quad \sec x = \frac{13}{-5}$$

Prove that each of the following equations is an identity. You must clearly show each step to get credit.

$$18. \frac{\sin 2\beta}{2\csc \beta} = \sin^2 \beta \cos \beta$$

$$\begin{aligned} \frac{\sin 2\beta}{2\csc \beta} &= (\sin 2\beta) \left(\frac{1}{2\csc \beta} \right) \\ &= (2\sin \beta \cos \beta) \left(\frac{1}{2} \sin \beta \right) \\ &= \sin^2 \beta \cos \beta \quad \checkmark \end{aligned}$$

$$19. \cos 3x = \cos x(1 - 4\sin^2 x)$$

$$\begin{aligned} \cos 3x &= \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x \\ &= (1 - 2\sin^2 x) \cos x - (2\sin x \cos x) \sin x \\ &= \cos x - 2\sin^2 x \cos x - 2\sin^2 x \cos x \\ &= \cos x - 4\sin^2 x \cos x \\ &= \cos x(1 - 4\sin^2 x) \quad \checkmark \end{aligned}$$

$$20. \cos\left(x - \frac{\pi}{2}\right) = \cos x \tan x$$

$$\begin{aligned} \cos\left(x - \frac{\pi}{2}\right) &= \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2} \\ &= \cos x(0) + \sin x(1) = \sin x \\ \cos x \tan x &= \cos x \left(\frac{\sin x}{\cos x} \right) = \sin x \quad \checkmark \end{aligned}$$

$$21. \frac{\sin y \cos y}{\tan y} = 1 - \sin^2 y$$

$$\begin{aligned} \frac{\sin y \cos y}{\tan y} &= \sin y \cos y \left(\frac{1}{\tan y} \right) = \sin y \cos y \cot y \\ &= \sin y \cos y \left(\frac{\cos y}{\sin y} \right) = \cos^2 y = 1 - \sin^2 y \quad \checkmark \end{aligned}$$

Use an appropriate identity to find the exact value of each expression.

$$22. \tan\left(\frac{-\pi}{12}\right)$$

$$\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \quad \text{OR} \quad \sqrt{3} - 2$$

$$23. \cos(105^\circ)$$

$$\frac{1 - \sqrt{3}}{2\sqrt{2}} \quad \text{OR} \quad \frac{\sqrt{2} - \sqrt{6}}{4}$$

Solve each problem.

$$24. \text{ Find the exact value of } \sin(2x) \text{ given that } \sin x = \frac{3}{5} \text{ and } x \text{ is in quadrant II.}$$

$$\sin(2x) = \frac{-24}{25}$$

Find the exact value of each expression.

25. $\sin^{-1}(-0.5)$

$$-30^\circ \text{ or } -\frac{\pi}{6}$$

26. $\cos^{-1}(-0.5)$

$$120^\circ \text{ or } \frac{2\pi}{3}$$

27. $\tan^{-1}(-1)$

$$-45^\circ \text{ or } -\frac{\pi}{4}$$

Find all real numbers in $[0, 2\pi]$ that satisfy each equation.

28. $2\cos(x) + 1 = 0$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

29. $2\sin(2x) + \sqrt{3} = 0$

$$x = \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}$$

30. $2\sin(-x) + \sqrt{3} = 0$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

31. $\sin\left(\frac{x}{2}\right) = 0$

$$x = 0, 2\pi$$

Find all real numbers in $[0, 360^\circ]$ that satisfy each equation.

32. $\sqrt{3}\tan\left(\frac{x}{2}\right) - \sqrt{3} = 0$

$$x = 90^\circ$$

33. $2\sin^2(x) + 1 = 3\sin x$

$$x = 30^\circ, 90^\circ, 150^\circ$$

34. $\sin^2(2x) - 1 = 0$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Find all real numbers that satisfy each equation. Answers should be in radians.

35. $\sin(x) - 1 = 0$

$$x = \frac{\pi}{2} + 2\pi k$$

for all integers, k

36. $\sin 2x = \tan x$

$$x = 0 + \pi k$$

for all integers, k

37. $2\cos(2x) + 1 = 0$

$$x = \begin{cases} \frac{\pi}{3} + \pi k \\ \frac{2\pi}{3} + \pi k \end{cases}$$

for all integers, k

38. $\cos 2x + \sin^2 x = 0$

$$x = \begin{cases} \frac{\pi}{2} + 2\pi k \\ \frac{3\pi}{2} + 2\pi k \end{cases}$$

State the number of possible triangles that can be formed using the given measurements.

39. $m\angle B = 123^\circ, a = 14\text{in}, b = 30\text{in}$

one triangle

40. $m\angle C = 67^\circ, b = 10\text{in}, c = 16\text{in}$

ONE TRIANGLE

41. $m\angle B = 47^\circ, a = 33\text{in}, b = 6\text{in}$

NO Triangle

42. $m\angle B = 32^\circ, a = 19\text{in}, b = 16\text{in}$

TWO TRIANGLES

Find each measurement indicated. Round your answer to the nearest tenth.

43. $m\angle A = 61^\circ, c = 14\text{in}, b = 18\text{in}$

Find a .

$$a = 16.6$$

44. $m\angle C = 17^\circ, a = 18\text{in}, b = 28\text{in}$

Find $m\angle A$

$$\angle A = 26.0$$

Solve the triangle. Round your answers to the nearest tenth.

45. $m\angle A = 123^\circ, a = 15\text{in}, c = 10\text{in}$

$$m\angle B = 23^\circ$$

$$m\angle C = 34^\circ$$

$$b = 6.99$$

46. $m\angle B = 109^\circ, a = 13\text{in}, b = 14\text{in}$

$$m\angle A = 61.4^\circ$$

$$m\angle C = 9.6^\circ$$

$$c = 2.5$$