NOTES: MATH 2 HONORS
Unit 10: Inscribed Quadrilaterals

Review Problem
A small car has a tire with a 13-inch diameter. A truck has a tire with a 29-inch diameter. How much farther than the car does the truck have to drive for its tire to complete one revolution?

Circumference: $2\pi r = \pi d$

small car: 13π in

truck: 29π in

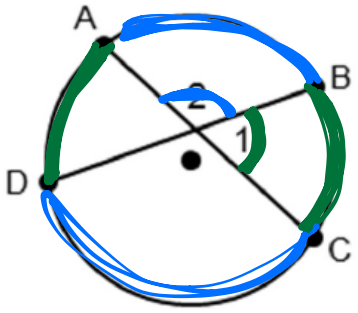
$29\pi - 13\pi = 16\pi$

≈ 50.3 in

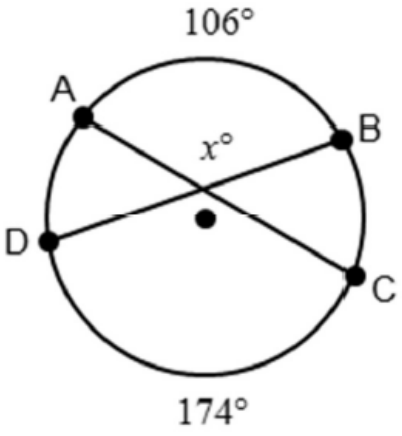
THEOREM:
If two chords intersect inside a circle, then the measure of each angle formed is half the sum of the measures of the arcs intercepted.

$$m\angle 2 = \frac{1}{2}\left(m\widehat{CD} + m\widehat{AB}\right)$$

$$m\angle 1 = \frac{1}{2}\left(m\widehat{BC} + m\widehat{AD}\right)$$



Example 1: Find the value of x.



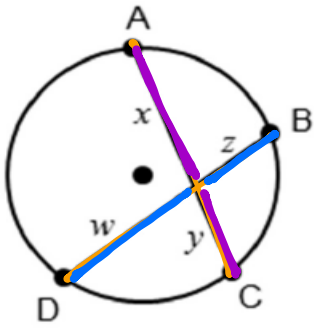
$x = \frac{1}{2}(174 + 106)$

$x = \frac{1}{2}(280)$

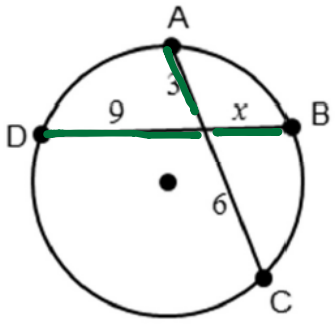
$x° = 140°$

THEOREM:
It two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

$$xy = wz$$



Example 2: Find the value of x.

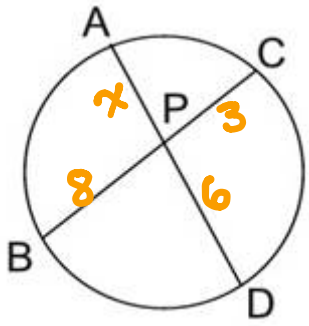


$9(x) = 3(6)$

$\frac{9x}{9} = \frac{18}{9}$

$x = 2$ units

Example 3: Find the value of x if $AP = x$, $PD = 6$, $BP = 8$, $PC = 3$.

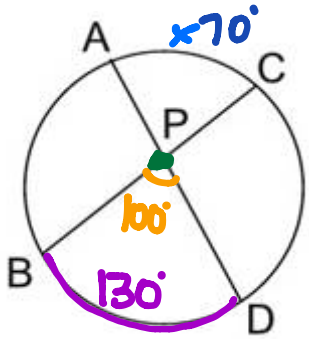


$6x = 8(3)$

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$ units

Example 4: Find $m\widehat{AC}$ if $m\widehat{BD} = 130°$ and $m\angle BPD = 100°$.



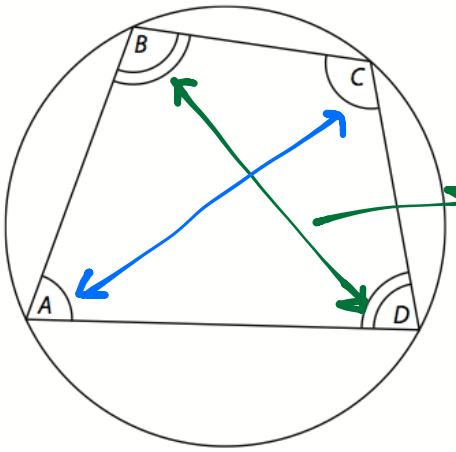
$2 \cdot 100 = \left(\frac{1}{2}(130 + x)\right)2$

$200 = 130 + x$

$x° = 70°$

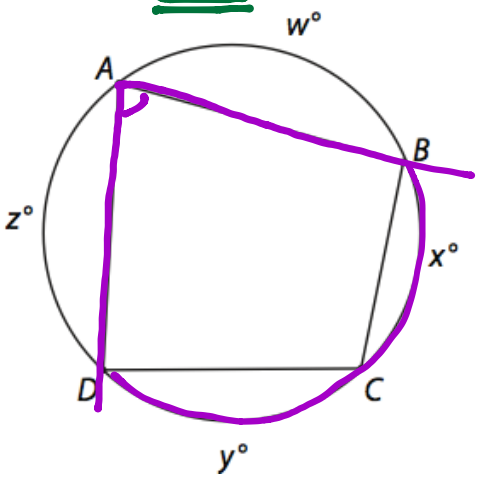
$m\widehat{AC} = 70°$

Inscribed Quadrilateral: A quadrilateral whose vertices are on a circle and the opposite angles on an inscribed quadrilateral are supplementary.



add to 180°

Example 5: Prove that the opposite angles of the given quadrilateral are supplementary.



$w^\circ + x^\circ + y^\circ + z^\circ = 360^\circ$

$\angle A = \frac{1}{2}(x^\circ + y^\circ)$ $\angle C = \frac{1}{2}(w^\circ + z^\circ)$

$\angle A + \angle C = \frac{1}{2}(x^\circ + y^\circ) + \frac{1}{2}(w^\circ + z^\circ)$

$\angle A + \angle C = \frac{1}{2}(x^\circ + y^\circ + w^\circ + z^\circ)$

$\angle A + \angle C = \frac{1}{2}(360) = 180$ ✓

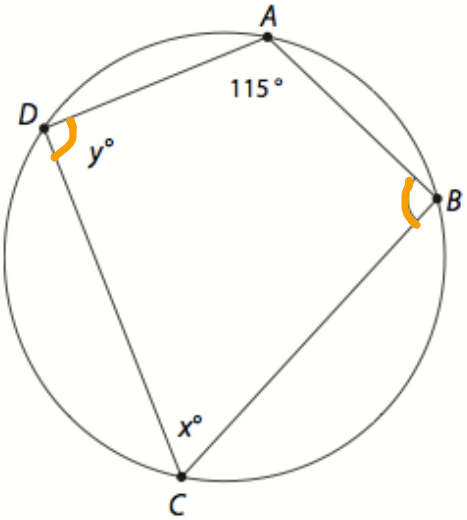
$\angle B = \frac{1}{2}(z^\circ + y^\circ)$ $\angle D = \frac{1}{2}(w^\circ + x^\circ)$

$\angle B + \angle D = \frac{1}{2}(z^\circ + y^\circ) + \frac{1}{2}(w^\circ + x^\circ)$

$\angle B + \angle D = \frac{1}{2}(z^\circ + y^\circ + w^\circ + x^\circ)$

$\angle B + \angle D = \frac{1}{2}(360) = 180$ ✓

Example 6: Find the values of x and y.



$x^\circ + 115^\circ = 180^\circ$

$x^\circ = 65^\circ$

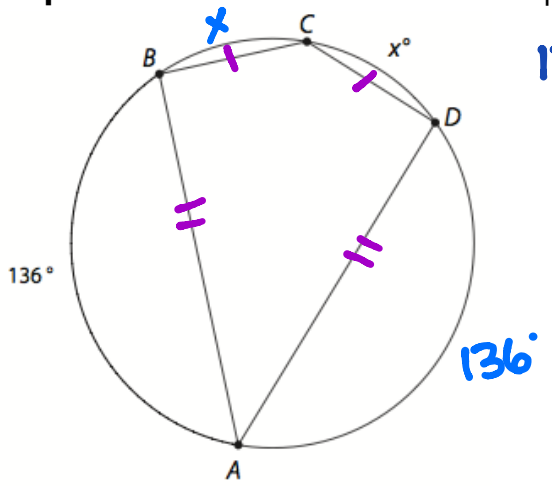
$\angle D + \angle B = 180^\circ$

$y^\circ + y^\circ = 180^\circ$

$2y^\circ = 180^\circ$

$y = 90^\circ$

Example 7: Find the values of x. Assume that quadrilateral $ABCD$ is a kite.



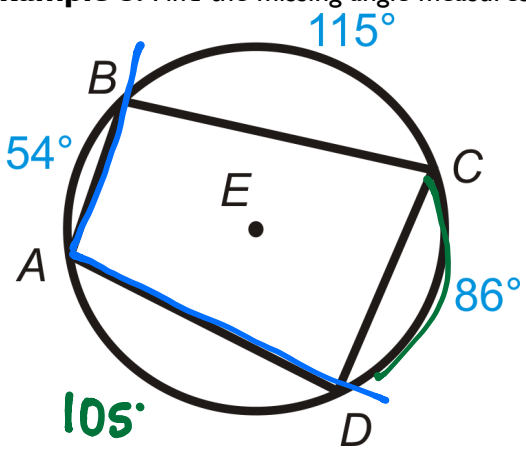
$136^\circ + 136^\circ + x^\circ + x^\circ = 360^\circ$

$272^\circ + 2x^\circ = 360^\circ$

$2x^\circ = 88^\circ$

$x^\circ = 44^\circ$

Example 8: Find the missing angle measures in quadrilateral $ABCD$.



$\angle A = \frac{1}{2}(115 + 86)$

$\angle A = 100.5^\circ$

$\angle C = 180 - \angle A = 180 - 100.5^\circ$

$\angle C = 79.5^\circ$

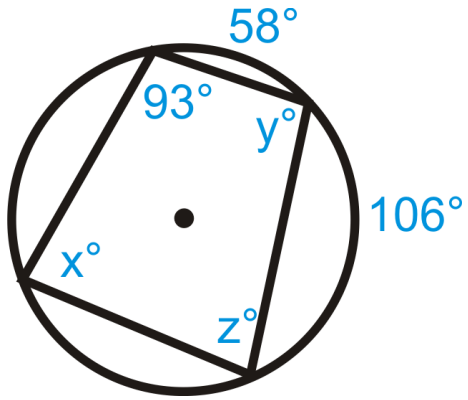
$\angle B = \frac{1}{2}(105 + 86)$

$\angle B = 95.5^\circ$

$\angle D = 180 - \angle B = 180 - 95.5^\circ$

$\angle D = 84.5^\circ$

Example 9: Find x, y, and z.



$z^\circ = 180 - 93^\circ$

$z^\circ = 87^\circ$

$x^\circ = \frac{1}{2}(58 + 106)$

$x^\circ = 82^\circ$

$y^\circ = 180^\circ - x^\circ = 180^\circ - 82^\circ$

$y^\circ = 98^\circ$