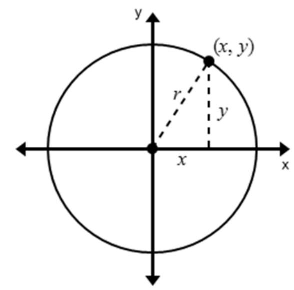
NOTES: MATH 2 HONORS
Unit 10: Equations of Circles

A circle is the set of all points equidistant from a given point called the center. In order to derive the equation of a circle, we need to find the distance from the center to any point (x, y) on the circle. This distance is the length of the radius. We can derive the equation of a circle using the distance formula or the Pythagorean Theorem.

Create a right triangle with the radius as the hypotenuse, the length *x* as the horizontal leg, and the length *y* as the vertical leg. Using the Pythagorean Theorem to relate the sides of the right triangle, we get the standard form of the **equation of a circle centered at the point** (h, k)**,** where *r* is the radius: $(x - h)^2 + (y - k)^2 = r^2$	

Example 1: Given the standard form of a circle, determine the center and the radius of the circle.

1. $(x - 2)^2 + (y - 3)^2 = 16$

center: (2,3)
radius: 4

2. $(x - 1)^2 + (y - 7)^2 = 25$

center: (1,7)
radius: 5

3. $x^2 + (y - 3)^2 = 4$

$(x+0)^2$
center: (0,3)
radius: 2

Example 2: Write the standard form of a circle with the given characteristics.

1. A circle with radius 10 and centered at $(8, -6)$.

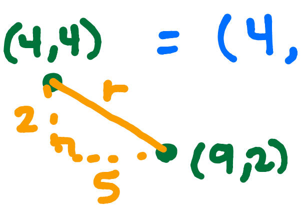
$(x-8)^2+(y+6)^2=100$

2. A circle with radius 5 and centered at $(-4,3)$.

$(x+4)^2+(y-3)^2=25$

3. A circle with endpoints of a diameter at $(9,2)$ and $(-1,6)$.

center: $\left(\frac{9+(-1)}{2}, \frac{2+6}{2}\right)$
$= (4, 4)$



$2^2+5^2=r^2$
$4+25=r^2$
$29=r^2$
$r=\sqrt{29}$

$\boxed{(x-4)^2+(y-4)^2=29}$

4. A circle with endpoints of a diameter at $(3,4)$ and $(-5,2)$.

Example 3: Find the center and radius of the circle described by the following equations..

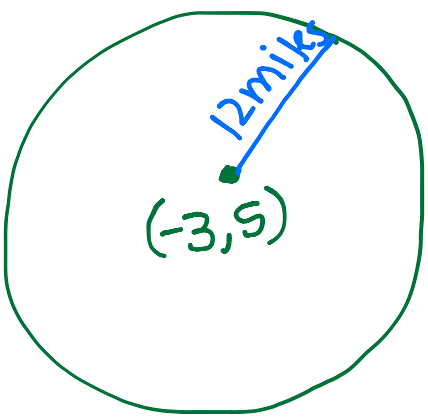
1. $x^2 + y^2 - 6x + 2y - 6 = 0$

$$x^2 - 6x + y^2 + 2y - 6 = 0$$
$$(x^2 - 6x + \underline{9}) + (y^2 + 2y + \underline{1}) - 6 - \underline{9} - \underline{1} = 0$$
$$\frac{-6}{2} = -3 \quad (-3)^2 = 9 \qquad \frac{2}{2} = 1 \quad (1)^2 = 1$$
$$(x-3)^2 + (y+1)^2 - 16 = 0$$
$$(x-3)^2 + (y+1)^2 = 16$$
$$\boxed{\text{center: } (3,-1) \quad \text{radius: } 4}$$

2. $4x^2 + 4y^2 + 20x - 40y + 116 = 0$

$$4x^2 + 20x + 4y^2 - 40y + 116 = 0$$
$$(4x^2 + 20x + \underline{\quad}) + (4y^2 - 40y + \underline{\quad}) + 116 - \underline{\quad} - \underline{\quad} = 0$$
$$4(x^2 + 5x + \underline{\tfrac{25}{4}}) + 4(y^2 - 10y + \underline{25}) + 116 - (4)\tfrac{25}{4} - (4)\underline{25} = 0$$
$$\frac{5}{2} = \frac{5}{2} \quad \left(\frac{5}{2}\right)^2 = \frac{25}{4} \qquad \frac{-10}{2} = -5 \quad (-5)^2 = 25$$
$$4\left(x + \frac{5}{2}\right)^2 + 4(y-5)^2 + 116 - 25 - 100 = 0$$
$$4\left(x + \frac{5}{2}\right)^2 + 4(y-5)^2 - 9 = 0$$
$$4\left(x + \frac{5}{2}\right)^2 + 4(y-5)^2 = 9$$
$$\left(x + \frac{5}{2}\right)^2 + (y-5)^2 = \frac{9}{4}$$
$$\boxed{\text{Center: } \left(-\frac{5}{2}, 5\right) \quad \text{radius: } \frac{3}{2}}$$

Example 4: A particular cell phone tower is designed to service a 12-mile radius. The tower is located at $(-3,5)$ on a coordinate plane whose units represent miles. What is the standard equation of the outer boundary of the region serviced by the tower? Is a cell phone user at $(8,0)$ within the service range? Explain.



$$\boxed{\text{Equation: } (x+3)^2 + (y-5)^2 = 144}$$

$$(8+3)^2 + (0-5)^2$$
$$= (11)^2 + (-5)^2$$
$$= 121 + 25 = 146$$

The cell phone user at (8,0) is out of the boundary because...