NOTES: MATH 2 HONORS
Unit 10: Introduction to Circles

Brainstorm: What do you know about a circle? Consider their properties, area, circumference, etc.

Since we know that **all circles are similar**, we can find the scale factor required to map one circle to another. For example, find the scale factor necessary to map ⊙A→⊙B, if:

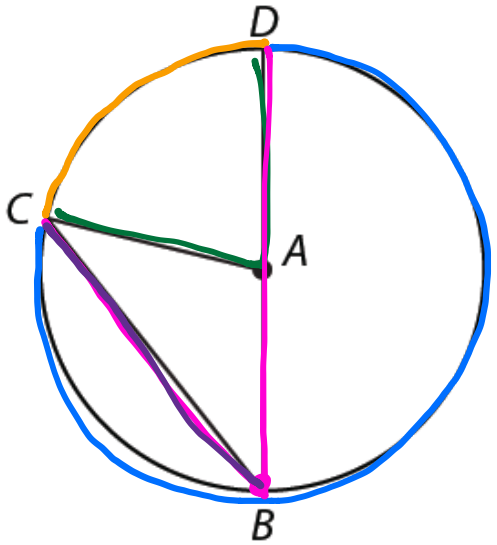
a. ⊙A has a radius of 4 units and ⊙B has a radius of 6 units.

$\frac{6}{4} = \frac{3}{2}$

b. ⊙A has a diameter of 55 units and ⊙B has a diameter of 75 units.

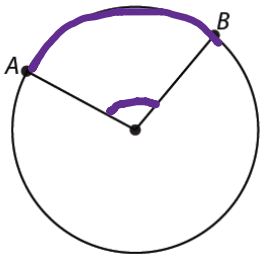
$\frac{75}{55} = \frac{15}{11}$

Use the diagram below to identify the following parts of the circle:

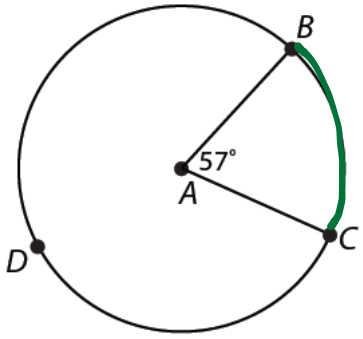


Part of Circle	Example(s)
Radius	$\overline{AC}$, $\overline{AD}$, $\overline{AB}$
Diameter	$\overline{DB}$
Minor Arc	$\overset{\frown}{DC}$, $\overset{\frown}{CB}$
Major Arc	$\overset{\frown}{CBD}$, $\overset{\frown}{CDB}$
Central Angle	$\angle DAC$, $\angle CAB$
Inscribed Angle	$\angle CBD$
Chord	$\overline{CB}$, $\overline{DB}$

Central Angle Theorem: The measure of a central angle is Congruent to the measure of its intercepted arc.



Example 1: Find the measure of the minor arc and the major arc.

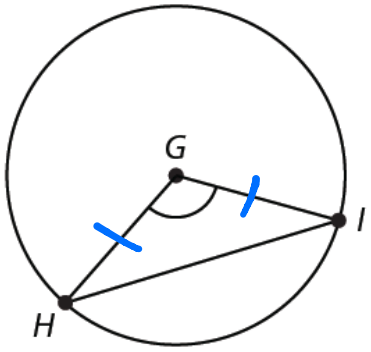


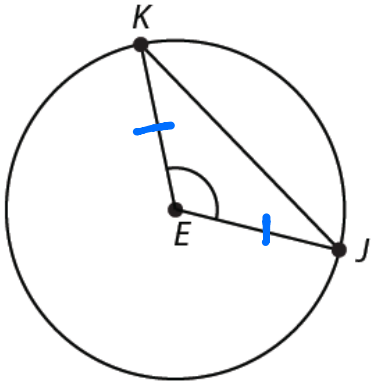
$\overset{\frown}{BC} = 57°$

$\overset{\frown}{BDC} = 360 - 57°$

$= 303°$

Example 2: $\odot G \cong \odot E$. What can you conclude about the chords? What can you conclude about the triangles?

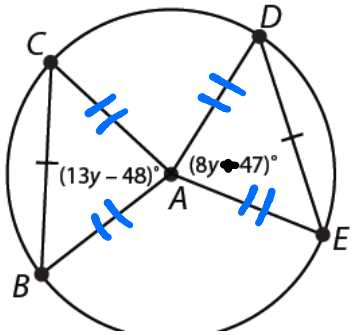




$\triangle GHI \cong \triangle EKJ$ by SAS since two corresponding sides are congruent and its included angle is congruent.

$\overline{HI} \cong \overline{KJ}$ by CPCTC

Example 3: Find y and find the measure of the central angle ∠CAB



$13y - 48 = 8y + 47$

$13y = 8y + 95$

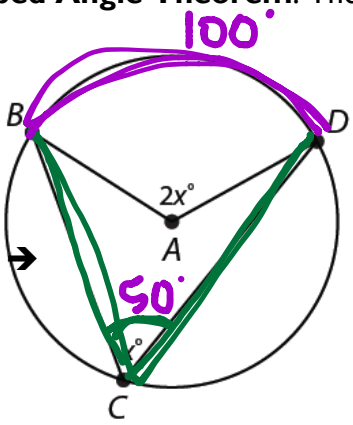
$5y = 95$

$y = 19$

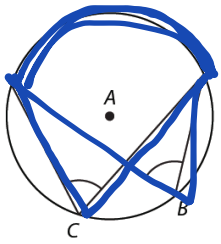
$m\angle CAB = 13(19) - 48$

$m\angle CAB = 199°$

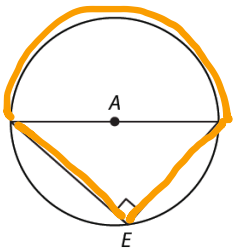
Inscribed Angle Theorem: The measure of an inscribed angle is half the measure of its intercepted arc



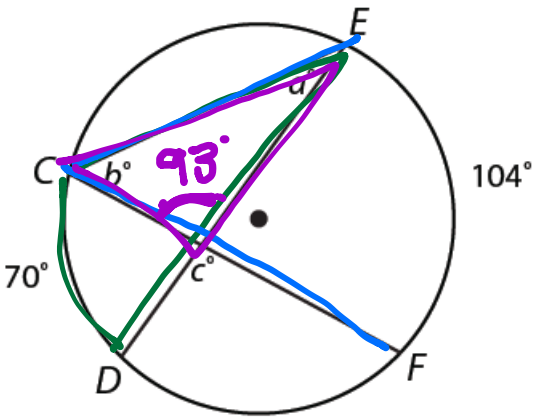
➔ Two inscribed angles that intercept the same arc are congruent



➔ An angle inscribed in a semicircle is a right angle.



Example 4: Find the value of each variable.



$a° = \frac{1}{2}(70)$

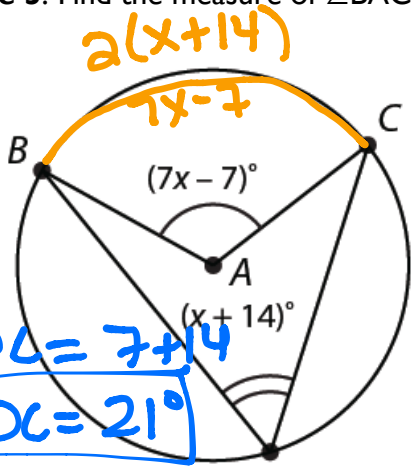
$a° = 35°$

$b° = \frac{1}{2}(104)$

$b° = 52°$

$c° = 93°$

Example 5: Find the measure of ∠BAC and ∠BDC (~~page 20~~)



$m\angle BDC = 7+14$

$m\angle BDC = 21°$

$m\angle BAC = 7(7) - 7$

$m\angle BAC = 42°$

$7x - 7 = 2(x+14)$

$7x - 7 = 2x + 28$

$+7 \quad +7$

$7x = 2x + 35$

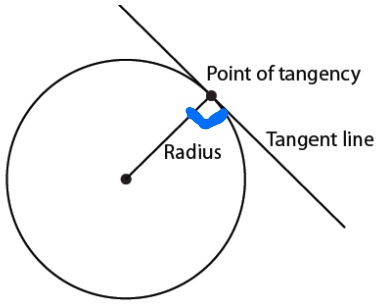
$-2x \quad -2x$

$5x = 35$

$\frac{5x}{5} = \frac{35}{5}$

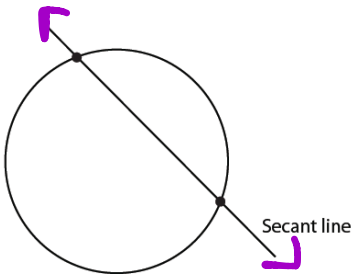
$x = 7$

Tangent Line: A line that intersects a circle at exactly one point.



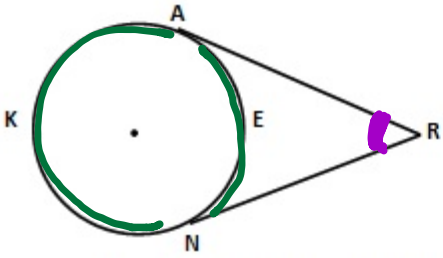
Tangent lines are perpendicular to the radius of the circle at the point of tangency.

Secant Line: A line that intersects a circle at two points.



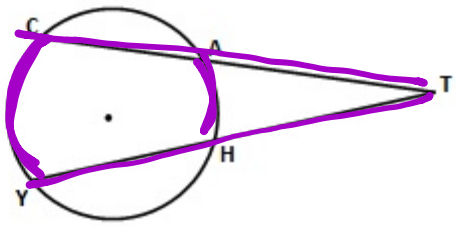
The following diagrams show the relationship between tangents, secants, angle measures, and arc measures in circles:

Two Tangents



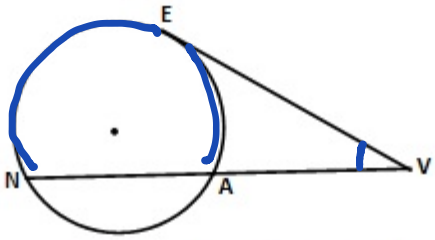
$m\angle ARN = \frac{1}{2}(m\widehat{NKA} - m\widehat{AEN})$

Two Secants



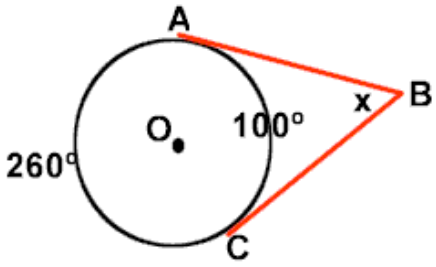
$m\angle ATH = \frac{1}{2}(m\widehat{CY} - m\widehat{AH})$

Tangent and Secant



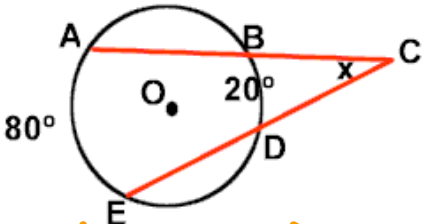
$m\angle EVA = \frac{1}{2}(m\widehat{EN} - m\widehat{EA})$

Numerical examples:



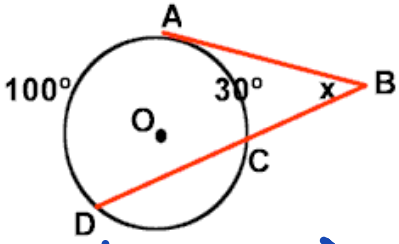
$x = \frac{1}{2}(260 - 100)$

$x = 80°$



$x = \frac{1}{2}(80 - 20)$

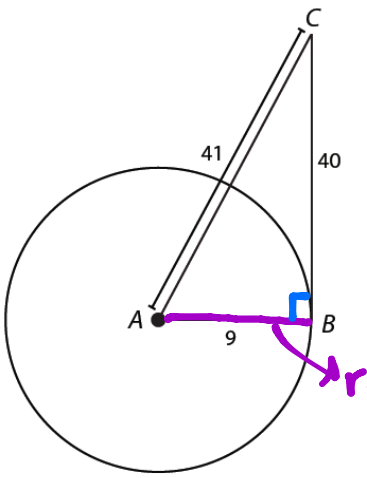
$x = 30°$



$x = \frac{1}{2}(100 - 30)$

$x = 35°$

Example 6: Determine whether $\overline{BC}$ is tangent to ⊙A.



$9^2 + 40^2 \stackrel{?}{=} 41^2$

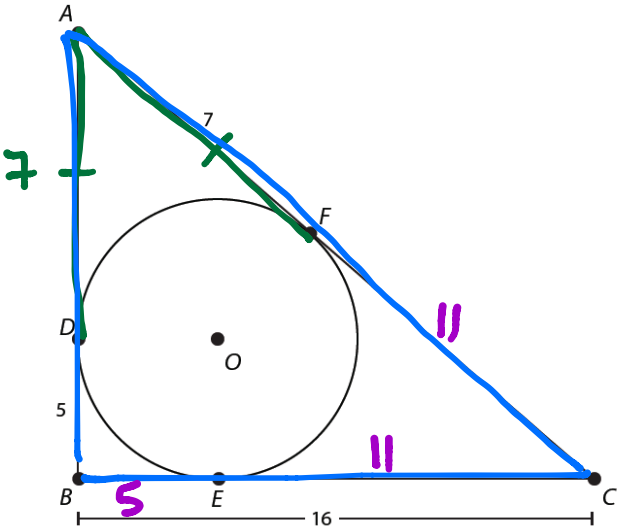
$81 + 1600 \stackrel{?}{=} 41^2$

$1681 = 1681$

radius

$\triangle ABC$ is a right triangle so $\overline{AB} \perp \overline{CB}$ and $\overline{AB}$ is the radius so $\overline{CB}$ has to be a tangent line.

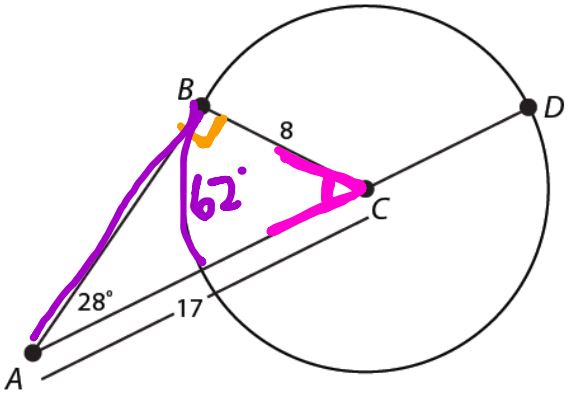
Example 7: Each side of ΔABC is tangent to circle O at the points D, E, and F. Find the perimeter of ΔABC.



$7 + 5 + 16 + 7 + 11$

$= 46$ units

Example 8: $\overline{AB}$ is tangent to ⊙C at point B as shown below.
Find the length of $\overline{AB}$ as well as the measure of arc BD.



$8^2 + (\overline{AB})^2 = 17^2$

$64 + (\overline{AB})^2 = 289$

$(\overline{AB})^2 = 225$

$\overline{AB} = 15$ units

$\angle BCA = 62°$

$28° = \frac{1}{2}(\overset{\frown}{BD} - 62)$

$28° = \frac{1}{2}\overset{\frown}{BD} - 31$

$+31 \quad +31$

$59 = \frac{1}{2}\overset{\frown}{BD}$

$\overset{\frown}{BD} = 118°$