NOTES: MATH 2 HONORS
Unit 10: Parabolas Part I

VOCAB:

- A **parabola** is the set of all points $P(x, y)$, in a plane that are an equal distance from both the **focus** (a fixed point) and the **directrix** (a fixed line).
- The **vertex** of a parabola is the high point or low point in the curve and is always halfway between the focus and the directrix. (The parabola and the directrix will never intersect.)

In previous units we discussed that parabolas either open up or down depending on the leading coefficient. Now, we are going to discuss when the parabola opens sideways.

Opens up or down with vertex (h, k)	**Opens left or right with vertex (h, k)**
Vertex form: $y = a(x-h)^2 + k$	**Vertex form:** $x = a(y-k)^2 + h$
"Conics" form: $4p(y-k) = (x-h)^2$ where p is the distance from the vertex to the focus/directrix. and $2p$ is the distance from the focus and the directrix.	**"Conics" form:** $4p(x-h) = (y-k)^2$ where p is the distance from the vertex to the focus/directrix. and $2p$ is the distance from the focus and the directrix.

** It is important to notice that h is always associated with the x, the k is always associated with the y, and p is always with the non-squared variable. **

Example 1: Determine the focus and the directrix for the given parabola. Is the parabola opening up, down, left, or right?

a. $2y = x^2$ open up

$2(y-0)=(x-0)^2$

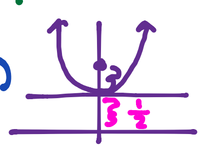
vertex: $(0,0)$

$4p=2$

$p=\frac{1}{2}$

Focus$=(0,\frac{1}{2})$

Directrix: $y=-\frac{1}{2}$



b. $24(y-8) = x^2$ opens up

$24(y-8)=(x-0)^2$

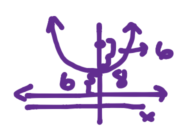
vertex: $(0,8)$

$4p=24$

$p=6$

Focus: $(0,14)$

Directrix: $y=2$



c. $y^2 = -48x$ open left

$(y-0)^2=-48(x-0)$

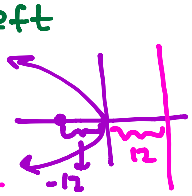
vertex: $(0,0)$

$4p=-48$

$p=-12$

Focus: $(-12,0)$

Directrix: $x=12$



d. $y^2 = \frac{1}{6}x$ open right

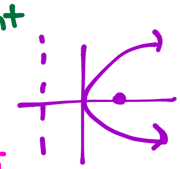
vertex: $(0,0)$

$4p=\frac{1}{6}$

$p=\frac{1}{24}$

Focus: $(\frac{1}{24},0)$

Directrix: $x=-\frac{1}{24}$



Example 2: Write the equation of the parabola with the given focus and vertex.

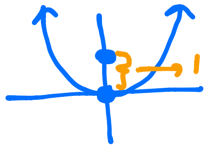
a. focus at $(0,1)$ and vertex at $(0,0)$

$p = 1$

$4p(y-k) = (x-h)^2$

$4(1)(y-0) = (x-0)^2$

$\boxed{4y = x^2}$



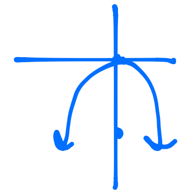
b. focus at $(0,-5)$ and vertex at $(0,0)$

$p = -5$

$4p(y-k) = (x-h)^2$

$4(-5)(y-0) = (x-0)^2$

$\boxed{-20y = x^2}$



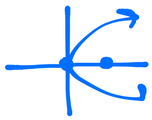
c. focus at $(2,0)$ and vertex at $(0,0)$ (h,k)

$p = 2$

$4p(x-h) = (y-k)^2$

$4(2)(x-0) = (y-0)^2$

$\boxed{8x = y^2}$



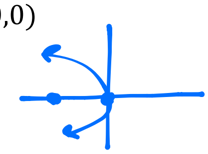
d. focus at $(-6,0)$ and vertex at $(0,0)$

$p = -6$

$4p(x-h) = (y-k)^2$

$4(-6)(x-0) = (y-0)^2$

$\boxed{-24x = y^2}$



Example 3: Write the equation of the parabola with the given directrix and vertex.

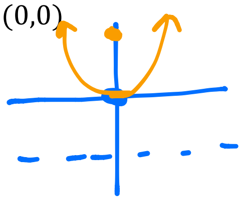
a. directrix at $y = -7$ and vertex at $(0,0)$

$p = 7$

$4p(y-k) = (x-h)^2$

$4(7)(y-0) = (x-0)^2$

$\boxed{28y = x^2}$



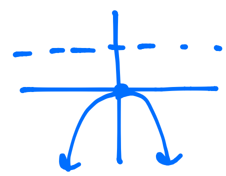
b. directrix at $y = 2$ and vertex at $(0,0)$

$p = -2$

$4p(y-k) = (x-h)^2$

$4(-2)(y-0) = (x-0)^2$

$\boxed{-8y = x^2}$



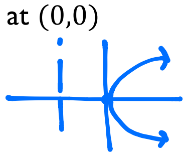
c. directrix at $x = -3$ and vertex at $(0,0)$

$p = 3$

$4p(x-h) = (y-k)^2$

$4(3)(x-0) = (y-0)^2$

$\boxed{12x = y^2}$



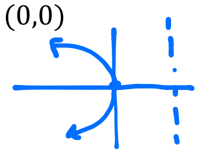
d. directrix at $x = 4$ and vertex at $(0,0)$

$p = -4$

$4p(x-h) = (y-k)^2$

$4(-4)(x-0) = (y-0)^2$

$\boxed{-16x = y^2}$



Distance Formula: $\sqrt{(x-x_1)^2+(y-y_1)^2}$

Example 4: Use the distance formula to find the equation of a parabola with focus $\overset{x_1,\,y_1}{(0,3)}$ and directrix $y=-3$

$$PF = PD$$

$$\sqrt{(x-x_1)^2+(y-y_1)^2} = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$\sqrt{(x-0)^2+(y-3)^2} = \sqrt{\cancel{(x-x)^2}+(y+3)^2}$$

$$\left(\sqrt{x^2+(y-3)^2}\right)^2 = \left(\sqrt{(y+3)^2}\right)^2$$

$$x^2+(y-3)^2 = (y+3)^2$$

$$x^2 = (y+3)^2-(y-3)^2$$

$$x^2 = (y^2+6y+9)-(y^2-6y+9)$$

$$\boxed{x^2 = 12y}$$

Example 5: Use the distance formula to find the equation of a parabola with focus $(-5,3)$ and directrix $y=9$

$$PF = PD$$

$$\sqrt{(x-(-5))^2+(y-3)^2} = \sqrt{(x-x)^2+(y-9)^2}$$

$$\left(\sqrt{(x+5)^2+(y-3)^2}\right)^2 = \left(\sqrt{(y-9)^2}\right)^2$$

$$(x+5)^2+(y-3)^2 = (y-9)^2$$

$$(x+5)^2 = (y-9)^2-(y-3)^2$$

$-18y+(+6y)$

$$(x+5)^2 = (y^2-18y+81)-(y^2-6y+9)$$

$$(x+5)^2 = -12y+72$$

$$\boxed{(x+5)^2 = -12(y-6)}$$

Example 6: Use the distance formula to find the equation of a parabola with focus $(2,0)$ and directrix $x = -2$

$$PF = PD$$

$$\sqrt{(x-2)^2+(y-0)^2} = \sqrt{(x+2)^2+(y-y)^2}$$

$$\left(\sqrt{(x-2)^2+y^2}\right)^2 = \left(\sqrt{(x+2)^2}\right)^2$$

$$(x-2)^2+y^2 = (x+2)^2$$

$$y^2 = (x+2)^2-(x-2)^2$$

$$y^2 = (x^2+4x+4)-(x^2-4x+4)$$

$$\boxed{y^2 = 8x}$$

Example 7: Use the distance formula to find the equation of a parabola with focus $(4,3)$ and directrix $x = 6$

$$PF = PD$$

$$\sqrt{(x-4)^2+(y-3)^2} = \sqrt{(x-6)^2+(y-y)^2}$$

$$\left(\sqrt{(x-4)^2+(y-3)^2}\right)^2 = \left(\sqrt{(x-6)^2}\right)^2$$

$$(x-4)^2+(y-3)^2 = (x-6)^2$$

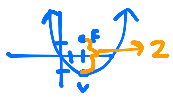
$$(y-3)^2 = (x-6)^2-(x-4)^2$$

$$(y-3)^2 = (x^2-12x+36)-(x^2-8x+16)$$

$$(y-3)^2 = -4x+20$$

$$\boxed{(y-3)^2 = -4(x-5)}$$

Focus: $(2,1)$ Vertex: $(2,-1)$



$p = 2$

$4p(y-k) = (x-h)^2$

$4(2)(y-(-1)) = (x-2)^2$

$\boxed{8(y+1) = (x-2)^2}$