Name ______________________ Period ________ Date ____________

Starter

1. Factor $9x^2 - 49$

$(3x-7)(3x+7)$

$(3x)^2 - (7)^2$

2. Factor $x^2 - 19x + 60$

$(x-15)(x-4)$

3. Simplify $5\sqrt[3]{32} - 6\sqrt[3]{4}$

$10\sqrt[3]{4} - 6\sqrt[3]{4} = 4\sqrt[3]{4}$

4. Simplify $\sqrt[3]{6} \cdot \sqrt[5]{6}$

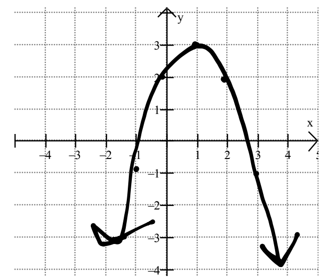
$6^{\frac{1}{3}} \cdot 6^{\frac{1}{5}} = 6^{\frac{1}{3}+\frac{1}{5}}$

$= 6^{\frac{5}{15}+\frac{3}{15}} = 6^{\frac{8}{15}}$

5. Fill out the table for the following function. $y = -(x-1)^2 + 3$

x	$f(x)$
-1	-1
0	2
1	3
2	2
3	-1

6. Use the table from #5 to graph the function.



2.2 Distinguishing Linear, Exponential, and Quadratic Relationships

For each function:

1st: Create a table of values.

2nd: Find the first difference and second difference from each table.

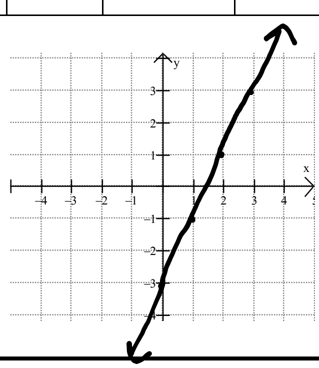
3rd: Graph the function.

4th: Identify characteristics of each function.

Linear

$f(x) = 2x - 3$

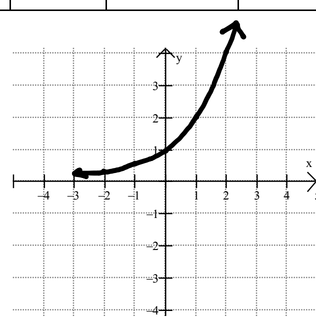
x	f(x)	1st Diff.	2nd Diff.
-2	-7	+2	0
-1	-5	+2	0
0	-3	+2	0
1	-1	+2	0
2	1	+2	
3	3		



Exponential

$f(x) = 2^x$

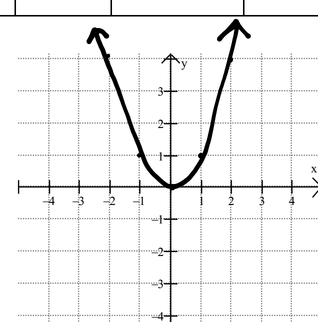
x	f(x)	1st Diff.	2nd Diff.
-2	1/4	+1/4	+1/4
-1	1/2	+1/2	+1/2
0	1	+1	+1
1	2	+2	+2
2	4	+4	
3	8		



Quadratic

$f(x) = x^2$

x	f(x)	1st Diff.	2nd Diff.
-2	4	-3	+2
-1	1	-1	+2
0	0	+1	+2
1	1	+3	+2
2	4	+5	
3	9		



In your own words, write a definition for the following vocabulary words:

1. Linear

2. Quadratic

3. Exponential

When determining if a function is linear, exponential, or quadratic, look at a table of values. (Note: The x-values must change by the same increment – aka they are the same distance apart.)

Linear	Quadratic	Exponential
1st common difference	2nd common difference	1st and 2nd difference is the same

EXAMPLE: Make a prediction on if the function is going represent a linear, exponential, or quadratic relationship. Prove your prediction by filling out a table and drawing a graph. Show your work on a separate piece of paper and staple it to this page.

at least 5 values

1. $y = 4x + 2$

2. $f(x) = -2^x$

3. $y = \frac{1}{2}x + 3$

4. $f(x) = 5 + 2^x$

5. $y = -x^2 - 1$

6. $y = 6$

7. $y = (x+1)(x-2)$

8. $f(x) = (x+2)^2$

9. $f(x) = (x+1)(x-3)(x+4)$

10. $y = \frac{2}{x} + 1$

This paper goes into your **NOTES** section of your binder!!

2.2: DISTINGUISH LINEAR, EXPONENTIAL, AND QUADRATIC RELATIONSHIPS

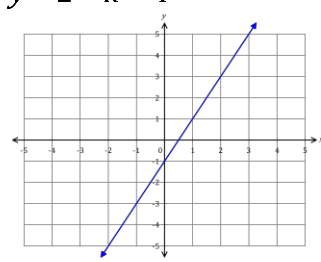
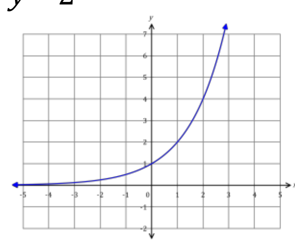
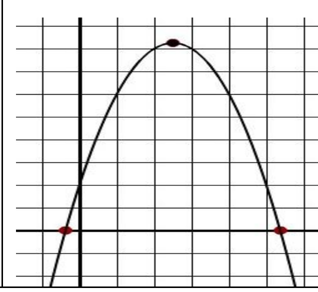
"I Can…" Statements:

- I can use a table or graph to observe that exponential functions grow more quickly than quadratic functions.
- I can distinguish linear, exponential, and quadratic relationships based on tables, equations, and verbal descriptions.

VOCABULARY

Line

exponent is the variable.

The variables in **linear functions** are only in the first degree, multiplied by constants, and combined only by addition and subtraction. Its graph is characterized by a line. Its slope is constant. Linear equations are called first-degree equations. The distance covered by a van traveling at a constant speed is a function of time. The relationship between time and distance is a **linear function.**	$2x + 3y = 6$ $4y = -7$ $f(x) = 2x - 3$ $2 + 3x = \frac{1}{2} - \frac{3}{4}x$ $y - 2 = x + 4$ 
An **exponential function** is one in which the variable occurs in the exponent. Its graph is characterized by a growth or decay graph. The value of an investment that grows at 4% per year is a function of the number of years. The relationship between the number of years and the value is an **exponential function.**	$y = 2^{-x}$ $y = 2^{x+3}$ $y = 2^x$ 
A **quadratic function** is of the form $ax^2 + bx + c = 0$, where *a, b,* and *c* are real numbers with $a \neq 0$. The highest variable in a quadratic function is of the second degree. Its graph is characterized by a "U"-shape called a **parabola.** Quadratic equations are called second-degree equations. No matter how hard you throw or kick a ball into the air, gravity returns it to Earth. How the height of a thrown ball changes over time is a **quadratic function.**	$x^2 - 2x - 8 = 0$ $3x + 10 = 4x^2$ $\frac{1}{2}x^2 + 2x - 6 = 0$ 

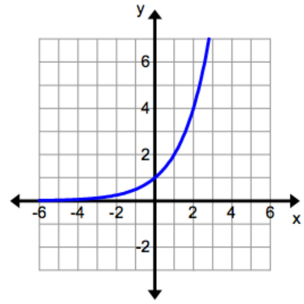
Linear functions can model *arithmetic* sequences, where the domain is the set of positive integers, because there is a **common difference** between each successive term. The common difference can also be called the **first difference.** Linear functions can model any pattern where the **first difference** between consecutive terms is the same number.

x	***y***
0	1
1	3
2	5
3	7

> + 2
> + 2
> + 2

Exponential Function- a function of the form $f(x) = ab^x$ where a and b are constants and $a \neq 0$, $b > 0$, and $b \neq 1$.

Exponential functions are most easily recognized by the variable in the exponent. The values of $f(x)$ are either increasing (exponential growth) if $a > 0$ and $b > 1$ or decreasing(exponential decay) if $a > 0$ and $0 < b < 1$.



$f(x) = 2^x$

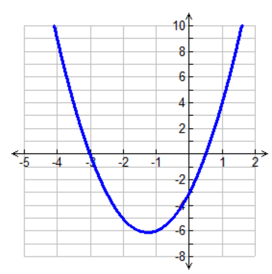
Exponential functions can model *geometric* sequences, where the domain is the set of positive integers, because each successive term is multiplied by the same number called the **common ratio**. **Exponential functions** can model any pattern where the next term is obtained by multiplying each successive term by the same number.

x	***y***
0	1
1	2
2	4
3	8

> x 2
> x 2
> x 2

Quadratic Function- a function that can be written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and a 0.

Quadratic functions are most easily recognized by the x^2 term. The graph is a **parabola**. A quadratic function can be formed by multiplying two **linear functions**. The quadratic function to the right can be written in factored form $f(x) = (2x - 1)(x + 3)$.



$f(x) = 2x^2 + 5x - 3$

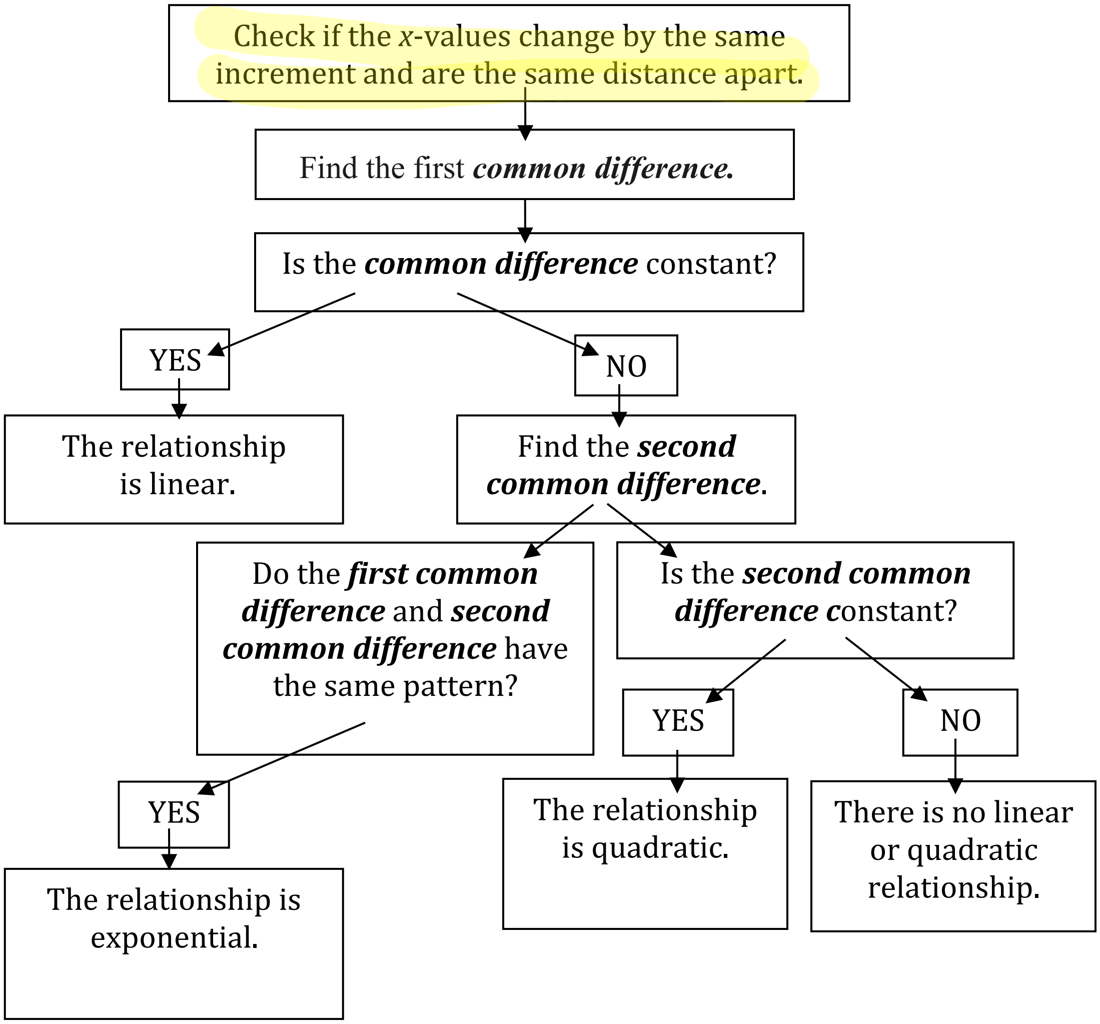
To determine if a pattern or a sequence can be modeled by a quadratic function, you have to look at the first and **second difference.** The **second difference** is the difference between the numbers in the first difference. If the first difference is not the same number but the **second difference** is, then the pattern or sequence can be modeled by a **quadratic function.**

x	*y*
-4	9
-3	0
-2	-5
-1	-6
0	-3
1	4

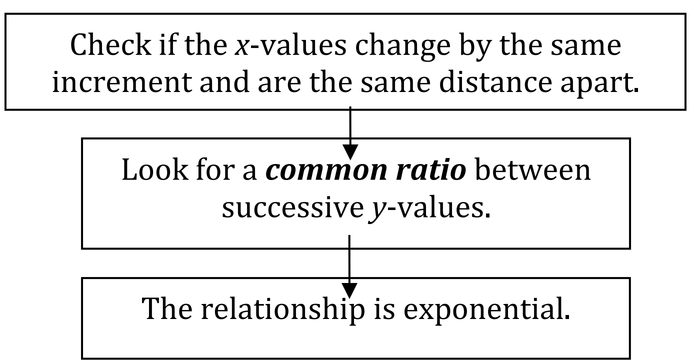
First difference: > -9, > -5, > -1, > +3, > +7

Second difference: > + 4, > + 4, > + 4, > + 4

HOW TO DETERMINE LINEAR, EXPONENTIAL, OR QUADRATIC RELATIONSHIPS USING COMMON DIFFERENCES OF A TABLE OF VALUES



HOW TO DETERMINE EXPONENTIAL RELATIONSHIPS USING A COMMON RATIO OF A TABLE OF VALUES



Example #1 

Gareth is saving money for a car. He has $16 in his bank account to start. Gareth puts $6 into his account each week. Complete the table for Gareth's bank account total below.

Tell whether the pattern is linear, quadratic, or exponential. Write a sentence to justify your conclusion. Write the equation that describes the relationship.

# of weeks (x)	Bank Account Total (y)	1st Difference	2nd Difference
0	16		
1	22	+6	
2	28	+6	
3	34	+6	
4	40	+6	
5	46	+6	

Answer: The *x*-values change by the same increment and are the same distance apart. The 1st common difference is constant + is linear.
Equation: The equation is $y = 6x$ 6. The relationship + 16.

Example #2
Tell whether the table represents a linear, quadratic, exponential relationship. Write a sentence to justify your conclusion.

x	y	1st Difference	2nd Difference
1	5		
2	8	+3	
3	11	+3	
4	14	+3	
5	17	+3	
6	20	+3	

Answer: The *x*-values change by the same increment and are the same distance apart. The 1st common difference is constant +3 so the relationship is linear.
Equation: $y = 3x + 2$. (The *y*-intercept is (0, 2).)

Example #3
Tell whether the table represents a linear, exponential, or quadratic relationship. Write a sentence to justify your conclusion.

COMMON RATIO METHOD:

x	y	Ratio
-10	3.5	$7 \div 3.5 = 2$
-5	7	$14 \div 7 = 2$
0	14	$28 \div 14 = 2$
5	28	$56 \div 28 = 2$
10	56	

Answer: The *x*-values change by the same increment and are the same distance apart. The common ratio is 2. The relationship is exponential.

FIRST AND SECOND COMMON DIFFERENCE PATTERN METHOD:

x	y	1st Difference	2nd Difference
-10	3.5	+3.5	+3.5
-5	7	+7	+7
0	14	+14	+14
5	28	+28	+28
10	56	+56	
15	112		

Answer: The *x*-values change by the same increment and are the same distance apart. The 1st common difference and the 2nd common difference pattern is the same. The relationship is exponential.

Example #4
Tell whether the table represents a linear, quadratic, or exponential function.

x	*y*
-2	-1
-1	0
0	1
1	3
2	9

Answer: There is no first common difference, second common difference, nor common ratio.

The relationship is none of these.

Example #5
Tell whether the table represents a linear, exponential, or quadratic relationship. Write a sentence to justify your conclusion.

x	f(x)
0	**0**
1	**6**
2	**14**
3	**24**
4	**36**
5	**50**

First, look at the *change* between the *y*-values. The change is not the same value

x	f(x)	1^{st} Difference
0	0	
		+6
1	6	
		+8
2	14	
		+10
3	24	
		+12
4	36	
		+14
5	50	

Second, look at the *change* between the first difference.

x	f(x)	1^{st} Difference	2^{nd} Difference
0	0		
		+6	
1	6		**+2**
		+8	
2	14		**+2**
		+10	
3	24		**+2**
		+12	
4	36		**+2**
		+14	
5	50		

Answer: The *x*-values are changing by the same increment and are the same distance apart. The 2^{nd} common difference is constant +2.
The relationship is quadratic.

Example #6

Tell whether the table represents a linear, exponential, or quadratic relationship. Write a sentence to justify your conclusion.

x	f(x)
-4	**5**
-2	**-3**
0	**-3**
2	**5**
4	**21**
6	**45**

First, look at the *change* between the *y*-values. The change is not the same value.

x	f(x)	1st Difference
-4	5	
-2	-3	**-8**
0	-3	**0**
2	5	**+8**
4	21	**+16**
6	45	**+24**

Second, look at the *change* between the first difference.

x	f(x)	1st Difference	2nd Difference
-4	5		
--2	-3	-8	
0	-3	0	**+8**
2	5	+8	**+8**
4	21	+16	**+8**
6	45	+24	**+8**

Answer: The *x*-values are changing by the same increment and are the same distance apart.

The 2nd common difference is constant +8. The relationship is quadratic.

Example #7

Tell whether each equation represents a linear, exponential, or quadratic relationship.

a. $f(x) = 3x + 2$ linear

b. $y = \frac{1}{2}x - 4$ linear

c. $x = 12$ linear

d. $y = 2x^2 - 3$ quadratic

e. $f(x) = \frac{2}{x} + 3$ none

f. $y = x(x + 2)$ quadratic

g. $f(x) = 2^x$ exponential

h. $y = (x - 2)(x + 4)$ quadratic

i. $f(x) = -2x + 4$ linear

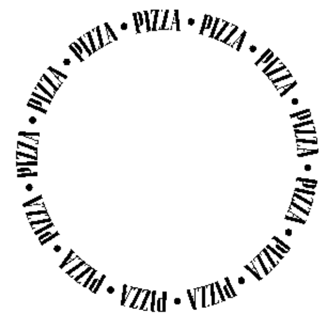
j. $f(x) = (x + 1)^2$ quadratic

k. $y = (x+3)(x-4)(x+1)$ none

l. $y = x^3 - x$ none

Practice Exercises A

1. Using a round pizza, find the maximum number of pizza pieces possible with six straight cuts through the center. Determine if this represents a linear, exponential, or quadratic relationship. Write the equation that describes the relationship.



# of cuts (x)	Total Pieces (y)	1st Difference	2nd Difference
1	2	+2	
2	4	+2	
3	6	+2	
4	8	+2	
5	10	+2	
6	12		

Answer:

Equation: $y = mx + b$

$y = 2x$

2. Stack of Cans

a. Complete the table below.

Stack Height *(x)*	# of Cans *(y)*	1st Difference	2nd Difference
1	1	+2	+1
2	3	+3	+1
3	6	+4	+1
4	10	+5	+1
5	15	+6	+1
6	21	+7	+1
7	28	+8	
8	36		

b. How many cans are in the bottom layer of a stack 8 layers high? 8 cans

c. Find the total number of cans in a stack 8 layers high. 36 cans

d. Does this table represent a linear, exponential, or quadratic relationship? Why?

Quadratic because the 2nd common difference is +1. AKA The 2nd difference is the same.

3. For each equation, complete the table and determine the 1st and 2nd common difference. Investigate the correlation between the *y*-values and the leading coefficient. Investigate the correlation between the leading coefficient and the 2nd difference. Describe what patterns you find.

a. $y = 2x^2$

x	y	1st Diff	2nd Diff
-2	8	-6	+4
-1	2	-2	+4
0	0	+2	+4
1	2	+6	
2	8		

b. $y = 3x^2$

x	y	1st Diff	2nd Diff
-2	12	-9	+6
-1	3	-3	+6
0	0	+3	+6
1	3	+9	
2	12		

c. $y = \frac{1}{2}x^2$

x	y	1st Diff	2nd Diff
-2	2	-1.5	+1
-1	$\frac{1}{2}$	-.5	+1
0	0	.5	+1
1	$\frac{1}{2}$	1.5	
2	2		

d. $y = -2x^2$

x	y	1st Diff	2nd Diff
-2	-8	+6	-4
-1	-2	+2	-4
0	0	-2	-4
1	-2	-6	
2	-8		

Patterns I found:

4. Complete the table and determine the 1st and 2nd common difference. Use what you discovered in question #3 to make predictions for each table. Write your predictions below.

a. $y = 5x^2$

x	y	1st Diff	2nd Diff
-2			
-1			
0			
1			
2			

b. $y = -4x^2$

x	y	1st Diff	2nd Diff
-2			
-1			
0			
1			
2			

c. $y = \frac{1}{4}x^2$

x	y	1st Diff	2nd Diff
-2	1		
-1	¼		
0	0		
1	¼		
2	1		

d. $y = ax^2$

x	y	1st Diff	2nd Diff
-2	4a		
-1	a		
0	0		
1	a		
2	4a		

Predictions:

Assignment 2.2

Name____Period________Date____________

1. Prove the tables below represent quadratic relationships by completing each table.

a.

x	y	1st Diff	2nd Diff
-3	10		
-2	5		
-1	2		
0	1		
1	2		
2	5		
3	10		

b.

x	y	1st Diff	2nd Diff
0	0		
1	3		
2	8		
3	15		
4			
5			
6			

c.

x	y	1st Diff	2nd Diff
-3	-9		
-2	-4		
-1	-1		
0	0		
1	-1		
2			
3			

d.

x	y	1st Diff	2nd Diff
0	-2		
1	0		
2	4		
3	10		
4			
5			
6			

Tell whether the pattern in each table is linear, exponential, exponential, or none of these. Support your answer.

2.

x	y
1	2
2	4
3	6
4	8
5	10
6	12
7	14

3.

x	y
0	1
1	9
2	24
3	729
4	2,187
5	6,561

4.

x	y
0	1
2	5
4	13
6	25
8	41
10	61
12	85

5.

x	y
0	1
1	5
3	13
6	25
8	41
10	61
12	85

6.

x	y
-3	12
-2	7
-1	4
0	3
1	4
2	7
3	12

7.

x	y
0	1
2	9
4	81
6	729
8	6561
10	59049
12	531441

8. Tell whether each equation represents a linear, exponential, or quadratic relationship.

a. $y = (x - 3)(x + 2)$

b. $y = 2^x$

c. $3x = 12$

d. $f(x) = x^2 - 2$

e. $y = 2x(x - 2)$

f. $y = x(x - 2)$

g. $f(x) = 3x + 2$

h. $y = \frac{1}{2}x - 4$

i. $f(x) = \frac{3}{x} + 2x$

j. $f(x) = 2x^3 + 2x$

9. Make a table of values for each equation below. Are the relationships linear, quadratic or exponential? Explain.

a. $y = 2^x$

x	y	1st diff	2nd diff
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			

b. $y = 3x - x^2$

x	y	1st diff	2nd diff
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			

c. $y = x - 2$

x	y	1st diff	2nd diff
-5	-7		
-4	-6		
-3	-5		
-2	-4		
-1	-3		
0	-2		
1	-1		
2	0		
3	1		
4	2		
5	3		

d. $y = x^2 + 5x + 6$

x	y	1st diff	2nd diff
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			

What cell phone company is best?

10. The total monthly charge for a cell phone at Long's Cell Phone Company is represented by the equation $y = x + 40$, where x represents the number of texts and y represents the total monthly charge.

The total monthly charge for a cell phone at the Save Cell Phone Company is represented by the equation $y = 2^x$, where x represents the number of texts and y represents the total monthly charge.

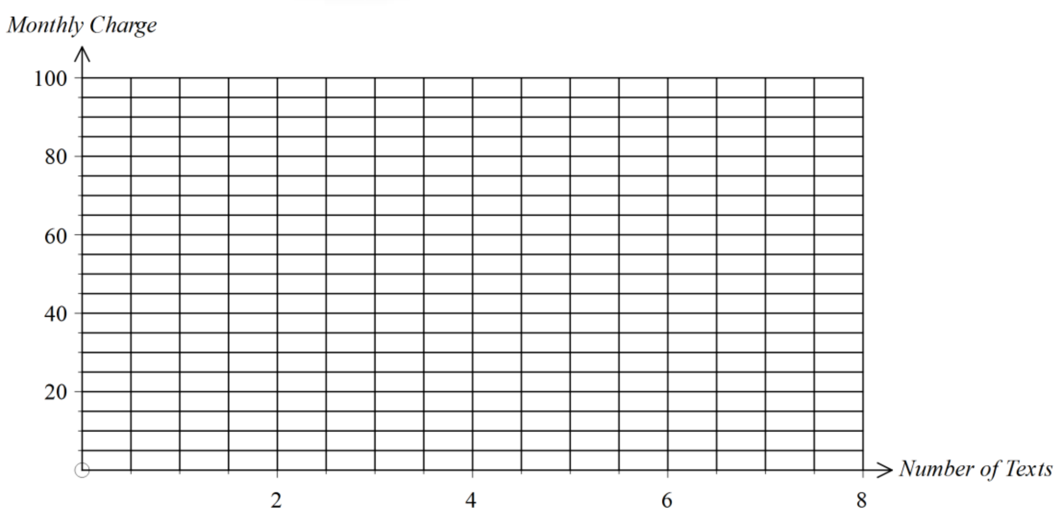
Represent each charge plan with a table. Graph each plan on the grid below.

Long's Cell Phone Company

x	*y*
0	
1	
2	
3	
4	
5	
6	

Save Cell Phone Company

x	*y*
0	
1	
2	
3	
4	
5	
6	



a. For each plan, tell whether the relationship between texts and monthly cost is linear, exponential, or quadratic. How do your equation, table, and graph support your answer?

b. Which cell phone company will you choose as your carrier? Why?

What stock option should she choose?

11. Rachel has the chance to buy stock in a video game company founded by her friend. She has played several of the video games developed by her friend and feels confident the company will be very successful.

She has $100 to invest and then has the choice of three options. The options' returns are shown in the table below.

Option A		**Option B**		**Option C**	
Years	**Return in $**	**Year(s)**	**Return in $**	**Year(s)**	**Return in $**
1	100	1	25	1	10
2	200	2	100	2	20
3	300	3	225	3	40
4	400	4	400	4	80
5		5		5	
6		6		6	
7		7		7	
8		8		8	
9		9		9	
10		10		10	

a. Complete the table above. Next, determine which option Rachel should choose for a long term investment. Justify your answer by showing the mathematics you applied to determine your choice.

b. Determine if each option represents a linear, exponential, or quadratic relationship.

Option A:

Option B:

Option C:

Selected Answers for Section 2.2

1. a

x	*y*	1st Diff	2nd Diff
-3	10		
-2	5	-5	
-1	2	-3	+2
0	1	-1	+2
1	2	+1	+2
2	5	+3	+2
3	10	+5	+2

4.

The *x*-values change by the same increment and are the same distance apart.

The second common difference is +4.

The relationship is quadratic.

8. e. quadratic h. linear j. none

9. a.

x	*y*	1st diff	2nd diff
-5	1/32	The *x*-values change by the same increment and are the same distance apart. The common ratio is ½. The equation is exponential.	
-4	1/16		
-3	1/9		
-2	¼		
-1	1/2		
0	1		
1	2		
2	4		
3	9		
4	16		
5	32		

9. d.

x	*y*	1st diff	2nd diff
-5	6	The *x*-values change by the same increment and are the same distance apart. The 2nd common difference is +2. The equation is quadratic.	
-4	2		
-3	0		
-2	0		
-1	2		
0	6		
1	12		
2	20		
3	30		
4	42		
5	56		