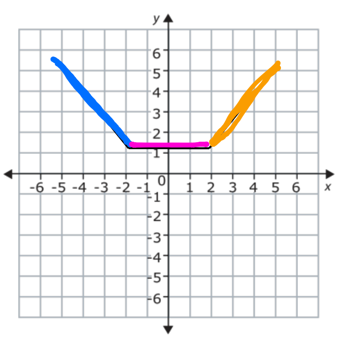
NOTES: SECONDARY 2 HONORS

IDENTIFY INTERVALS AND END BEHAVIOR (2.6) and CALCULATE RATE OF CHANGE (2.10)

Vocabulary

- An **interval** is a set of numbers between two x-values
  - An **open interval** does not include the end values
    - In *interval notation*, open intervals are represented with parenthesis (,)
    - In *inequality notation*, open intervals are represented with $<$, $>$
    - On *number lines*, open intervals are represented with open circles o
  - A **closed interval** does include the end values
    - In *interval notation*, closed intervals are represented with brackets [,]
    - In *inequality notation*, closed intervals are represented with $\le$, $\ge$
    - On *number lines*, closed intervals are represented with closed circles •
  - When an interval extends indefinitely in one direction, we use the special symbol:
    - In interval notation, "**infinity**" is always accompanied by an open parenthesis
- **Interval Notation** is used:
  - to describe the domain and range of functions
  - to describe where functions are increasing, decreasing, or **constant**
  - to describe where functions have positive or negative y-values

Example #1:



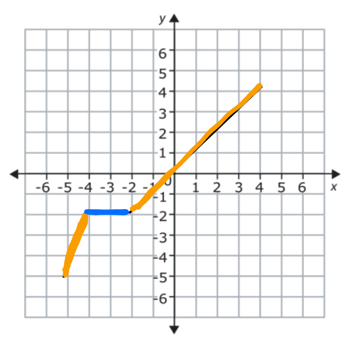
The function is **increasing** on the interval: (2, 5)

The function is **decreasing** on the interval: (-5, -2)

The function is **constant** on the interval: (-2, 2)

The function is **positive** on the interval: (-5, 5)

Example #2:



The function is **increasing** on the interval(s): (-5, -4) U (-2, 4)

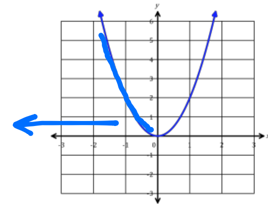
The function is **constant** on the interval: (-4, -2)

The function is **positive** on the interval: (0, 4)

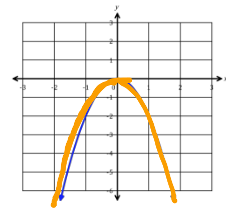
The function is **negative** on the interval: (-5, 0)

Vocabulary:

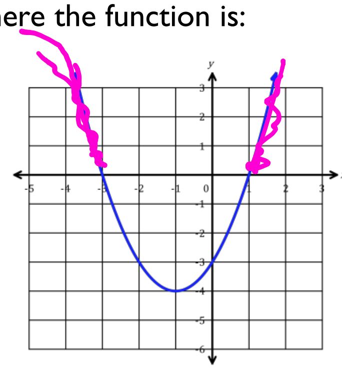
- **End behavior** describes the y-values of a graph as the x-values approach $-\infty$ and ∞
  - If the leading coefficient of a quadratic function is positive, the end behavior is described as:
    - Left End Behavior: $x \to -\infty$ $y \to \infty$
    - Right End Behavior: $x \to \infty$ $y \to \infty$



  - If the leading coefficient of a quadratic function is negative, the end behavior is described as:
    - Left End Behavior: $x \to -\infty$ $y \to -\infty$
    - Right End Behavior: $x \to \infty$ $y \to -\infty$



Example #3: Given the graph of $f(x) = x^2 + 2x - 3$, find the intervals where the function is:



a) Increasing $(-1, \infty)$

b) Decreasing $(-\infty, -1)$

c) Constant NA

d) Positive $(-\infty, -3) \cup (1, \infty)$

e) Negative $(-3, 1)$

f) Determine the end behavior of the graph.
Left hand: $x \to -\infty$ $y \to \infty$
Right hand: $x \to \infty$ $y \to \infty$

Example #4: Given the quadratic function shown in the table, find the intervals where the function is:

a) Increasing $(-\infty, 2)$

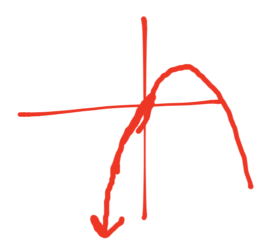
b) Decreasing $(2, \infty)$

c) Constant NA

d) Positive $(0, 4)$

e) Negative $(-\infty, 0) \cup (4, \infty)$

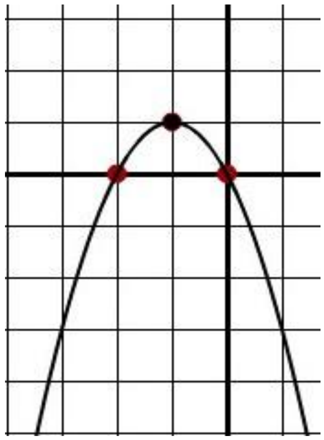
x	***y***
-2	-12
-1	-5
0	0
1	3
2	4
3	3
4	0
5	-5



f) Determine the end behavior of the function.
Left hand: $x \to -\infty$ $y \to -\infty$
Right hand: $x \to \infty$ $y \to -\infty$
* Leading coefficient: negative *

More Practice:

a)



x-intercept(s): (-2,0) & (0,0)
y-intercept: (0,0)

maximum or minimum: max value at 1 when x is -1.

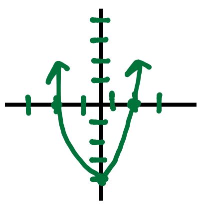
increasing interval: $(-\infty,-1)$
decreasing interval: $(-1,\infty)$

positive interval(s): $(-2,0)$
negative interval(s): $(-\infty,-2) \cup (0,\infty)$

left end behavior: $x \to -\infty$ $y \to -\infty$
right end behavior: $x \to \infty$ $y \to -\infty$

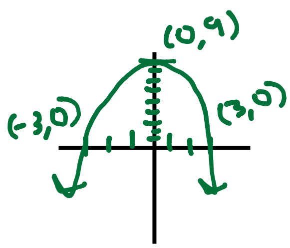
Graph each function using a graphing calculator. Find the x- and y-intercept(s), máximum or minimum (vertex), increasing, decreasing, positive and negative intervals, and end behavior. Label your graph.

b) $f(x) = x^2 - 4$



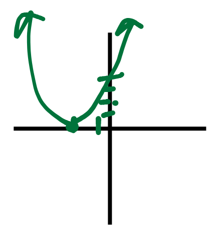
x-intercept(s): (-2,0) & (2,0)
y-intercept: (0,-4)
max/min: min value at -4 when x is 0.
increasing: $(0,\infty)$
decreasing: $(-\infty,0)$
positive: $(-\infty,-2) \cup (2,\infty)$
negative: $(-2,2)$
left end behavior:
as $x \to -\infty$, $y \to \infty$
right end behavior:
as $x \to \infty$, $y \to \infty$

c) $h(x) = -x^2 + 9$



x-intercept(s): (-3,0) & (3,0)
y-intercept: (0,9)
max/min: max value at 9 when x is 0
increasing: $(-\infty,0)$
decreasing: $(0,\infty)$
positive: $(-3,3)$
negative: $(-\infty,-3) \cup (3,\infty)$
left end behavior:
as $x \to -\infty$, $y \to -\infty$
right end behavior:
as $x \to \infty$, $y \to -\infty$

d) $y = (x + 2)^2$



x-intercept(s): (-2,0)
y-intercept: (0,4)
max/min: min value at 0 when x is -2
increasing: $(-2,\infty)$
decreasing: $(-\infty,-2)$
positive: $(-\infty,\infty)$
negative: NA
left end behavior:
as $x \to -\infty$, $y \to \infty$
right end behavior:
as $x \to \infty$, $y \to \infty$

e)

x	*y*
-5	0
-4	-3
-3	-4
-2	-3
-1	0
0	5

x-intercept(s): (-5,0) & (-1,0)
y-intercept: (0,5)
max/min: min value at -4 when x is -3.
increasing: $(-3,\infty)$
decreasing: $(-\infty,-3)$
positive: $(-\infty,-5) \cup (-1,\infty)$
negative: $(-5,-1)$
left end behavior:
as $x \to -\infty$, $y \to \infty$
right end behavior:
as $x \to \infty$, $y \to \infty$

leading coefficient: positive

f)

x	*y*
-2	0
-1	-1
0	0
1	3
2	8
3	15
4	24
5	35

x-intercept(s): (-2,0) & (0,0)
y-intercept: (0,0)
max/min: min value at -1 when x is -1.
increasing: $(-1,\infty)$
decreasing: $(-\infty,-1)$
positive: $(-\infty,-2) \cup (0,\infty)$
negative: $(-2,0)$
left end behavior:
as $x \to -\infty$, $y \to \infty$
right end behavior:
as $x \to \infty$, $y \to \infty$

leading coefficient: positive

Vocabulary:

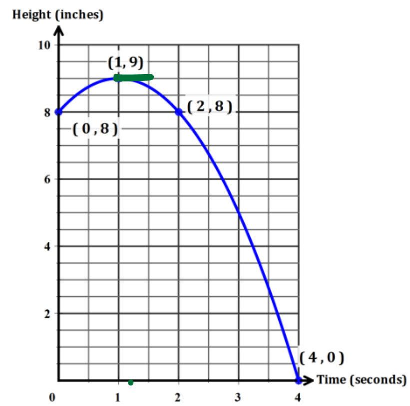
- The **rate of change** describes the rate at which one quantity is changing with respect to another quantity.
- The **average rate of change** is a rate of change (or slope) over an interval.
    - The formula for calculating the **average rate of change** is:

$$\frac{f(b)-f(a)}{b-a} \qquad \frac{\text{change in } y}{\text{change in } x} \qquad \frac{\Delta y}{\Delta x}$$

- The line passing through the two endpoints of the interval is referred to as the **secant line**

Example #1: $h(t)$ represents the height of an object, in *inches*, tossed up in the air. Find the average rate of change of the height of the object on the given time intervals.

$h(t) = \quad t^2 + 2t + 8$



[0, 1], [1, 2], [2, 4], and [0, 2]

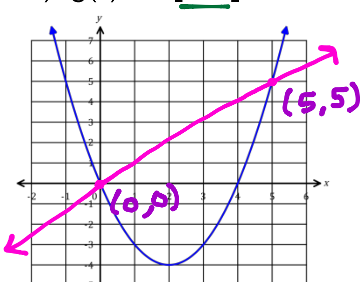
$[0,1]: \frac{f(1)-f(0)}{1-0} = \frac{9-8}{1-0} = \frac{1}{1} \quad \boxed{1 \text{ in/sec}}$

$[1,2]: \frac{f(2)-f(1)}{2-1} = \frac{8-9}{2-1} = \frac{-1}{1} \quad \boxed{-1 \text{ in/sec}}$

Example #2: For each of the following, draw the **secant line** that connects the two end points. Write the coordinates of the two end points, and then calculate the **average rate of change** on the specified interval.

$g(x) = x^2 - 4x$

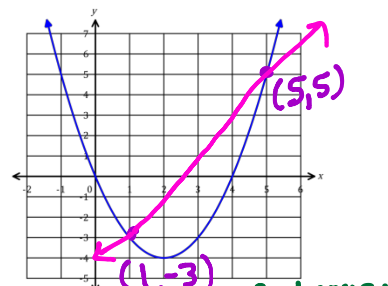
a) $g(x)$ on [0, 5]



Average rate of change:

$\frac{f(5)-f(0)}{5-0} = \frac{5-0}{5-0} = \frac{5}{5} = \boxed{1}$

b) $g(x)$ on [1, 5]

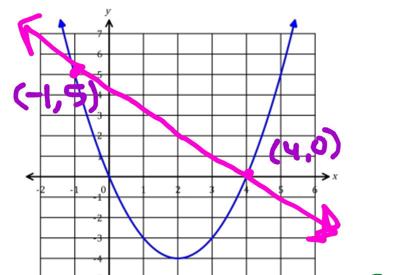


average rate of change:

$\frac{g(5)-g(1)}{5-1} = \frac{5-(-3)}{5-1} = \frac{8}{4} = 2$

$= \boxed{2}$

c) $g(x)$ on [-1, 4]



average rate of change:

$\frac{g(4)-g(-1)}{4-(-1)} = \frac{0-5}{4+1} = \frac{-5}{5}$

$= \boxed{-1}$

Example #3: The per capita consumption of ready-to-eat and ready-to-cook breakfast cereal is shown below. Find the average rate of change from 1992 to 1995 and interpret its meaning.

Years since 1990	0	1	2	3	4	5	6	7	8	9
Cereal Consumption (pounds)	15.4	16.1	16.6	17.3	17.4	17.1	16.6	16.3	15.6	15.3

$\frac{f(5)-f(2)}{5-2} = \frac{17.1-16.6}{5-2} = \frac{.5}{3} \approx \boxed{0.1\overline{6} \text{ lb/year}}$