NOTES: SECONDARY 2 HONORS

COMPARING QUADRATIC, LINEAR, AND EXPONENTIAL FUNCTIONS (2.11)

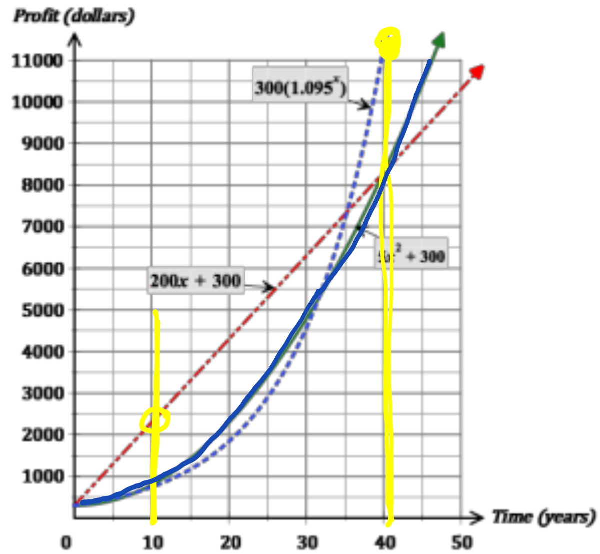
Example 1: Comparing Rates of Change Using Equations

The graph on the right shows three different investment plans that model the following functions:

$\boxed{L(t)} = 200t + 300$

$Q(t) = 5t^2 + 300$

$E(t) = 300(1.095^t)$



a. If you were going to make an investment of $300, and planned to withdraw your money after 10 years, which investment plan would you choose? Use the **average rate of change** to support your answer.

Pick L(t) because...

$L(t): \frac{L(10)-L(0)}{10-0} = \frac{2300-300}{10-0} = \frac{2000}{10} = \$200$

$Q(t): \frac{Q(10)-Q(0)}{10-0} = \frac{800-300}{10-0} = \frac{500}{10} = \$50$

$E(t) = \frac{E(10)-E(0)}{10-0} = \frac{743.47-300}{10-0} = \frac{443.47}{10} = \$44.35$

b. If you were going to make an investment of $300, and planned to withdraw your money after 40 years, which investment plan would you choose? Use the **average rate of change** to support your answer.

Pick E(t) because...

$L(t): \frac{L(40)-L(0)}{40-0} = \frac{8300-300}{40-0} = \frac{8000}{40} = \$200$

$Q(t): \frac{Q(40)-Q(0)}{40-0} = \frac{8300-300}{40-0} = \frac{8000}{40} = \$200$

$E(t): \frac{E(40)-E(0)}{40-0} = \frac{11315.82-300}{40-0} = \frac{11015.82}{40} \approx \$275.40$

c. Would you ever choose to invest your money in the quadratic investment plan, Q(t)? Why or why not?

NO, because....

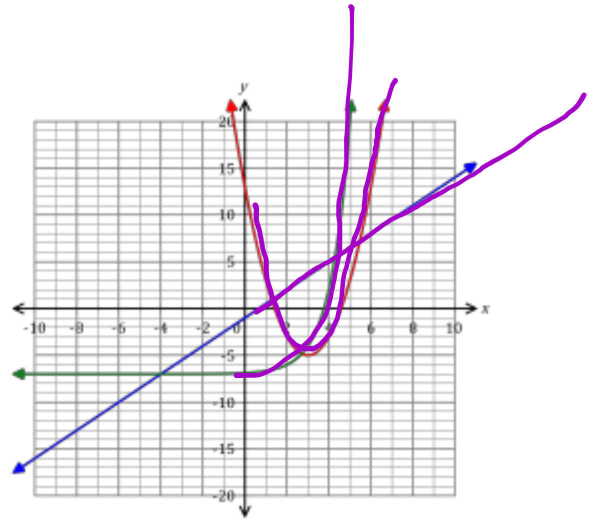
Example 2: Compare Rates of Change Using Graphs

a. Using a graphing calculator, graph the following functions on the same coordinate plane.

$L(x) = \frac{3}{2}x - 1$

$E(x) = 3^{x-2} - 7$

$Q(x) = 2(x-3)^2 - 5$



b. Find the average rate of change for the functions $L(x)$, $E(x)$, and $Q(x)$ for the specified intervals. Determine which of the three functions is increasing the fastest on each interval.

[-4, 2]

$L(x)$: $\frac{L(2)-L(-4)}{2-(-4)} = \frac{2-(-7)}{2-(-4)} = \frac{9}{6} = \frac{3}{2}$

$E(x)$: $\frac{E(2)-E(-4)}{2-(-4)} = \frac{-6-(-7)}{2-(-4)} = \frac{1}{6}$

$Q(x)$: $\frac{Q(2)-Q(-4)}{2-(-4)} = \frac{(-3)-93}{2-(-4)} = \frac{-96}{6} = -16$

L(x) is increasing the fastest on the interval [-4, 2].

[3, 5]

$L(x)$: $\frac{L(5)-L(3)}{5-3} = \frac{6.5-3.5}{5-3} = \frac{3}{2}$

$E(x)$: $\frac{E(5)-E(3)}{5-3} = \frac{20-(-4)}{5-3} = \frac{24}{2} = 12$

$Q(x)$: $\frac{Q(5)-Q(3)}{5-3} = \frac{3-(-5)}{5-3} = \frac{8}{2} = 4$

E(x) is increasing the fastest on the interval [3, 5].

c. Which function has the greatest average rate of change on the interval $[0, \infty)$?

E(x) has the greatest average rate of change on $[0, \infty)$ because as x grows larger, the y-values of E(x) grow larger than L(x) and Q(x).

HW: Example 3 in Notes
pg. 219 : 1-2
write (and answer) two test questions from unit 2.

Example 3: Compare Rates of Change Using Tables

a. Complete the tables:

x	$f(x)=2x$
-2	-4
-1	-2
0	0
1	2
2	4
3	6
4	8
5	10

x	$g(x)=x^2$
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25

x	$h(x)=2^x$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32

b. Find the average rate of change for the functions $f(x)$, $g(x)$, and $h(x)$ for the specified intervals. Determine which of the three functions is increasing the fastest on the interval.

[0, 1]

$$\frac{f(0)-f(1)}{0-1}=\frac{0-2}{0-1}=\frac{-2}{-1}=2$$

$$\frac{g(0)-g(1)}{0-1}=\frac{0-1}{0-1}=\frac{-1}{-1}=1$$

$$\frac{h(0)-h(1)}{0-1}=\frac{1-2}{0-1}=\frac{-1}{-1}=1$$

f(x) is increasing the fastest over interval [0,1].

[3, 5]

$$\frac{f(5)-f(3)}{5-3}=\frac{10-6}{2}=\frac{4}{2}=2$$

$$\frac{g(5)-g(3)}{5-3}=\frac{25-9}{2}=\frac{16}{2}=8$$

$$\frac{h(5)-h(3)}{5-3}=\frac{32-27}{2}=\frac{5}{2}$$

g(x) is increasing the fastest over the interval [5,3].

[-2, 5]

$$\frac{f(5)-f(-2)}{5-(-2)}=\frac{10-(-4)}{7}=\frac{14}{7}=2$$

$$\frac{g(5)-g(-2)}{5-(-2)}=\frac{25-4}{7}=\frac{21}{7}=3$$

$$\frac{h(5)-h(-2)}{5-(-2)}=\frac{32-\frac{1}{4}}{7}=\frac{31.25}{4}\approx 7.81$$

h(x) is increasing the fastest over the interval [-2,5].

Making a Generalization:

What type of function (linear, exponential, or quadratic) will increase fastest as $x \to \infty$? Explain your reasoning.