NOTES: SECONDARY 2 HONORS

BUILDING FUNCTIONS THAT MODEL RELATIONSHIPS BETWEEN TWO QUANTITIES (3.2)

Vocabulary

- A **function** is a relation for which each input has exactly one output. If any input has more than one output, then the relation is **not** a function.
    - We can use the Vertical Line Test to determine if a graphed relation is a function.
- *f(x)* is a fancy way of writing y

COMBINING FUNCTIONS USING ARITHMETIC OPERATIONS

Let *f* and *g* be any two functions. A new function *h* can be created by performing any of the the four basic operations on *f* and *g*. The domain of *h* consists of the x-values that are in the domains of both *f* and *g*. Additionally, the domain of a quotient does not include *x*-values for which $g(x) = 0$.

Example 1: $f(x) = 5x^2 + 2x;\ g(x) = -3x^2$

a. Find $h(x) = f(x) + g(x)$.

Operation: addition

$h(x) = (5x^2 + 2x) + (-3x^2)$

$\boxed{h(x) = 2x^2 + 2x}$

b. Find $h(x) = f(x) - g(x)$.

Operation: subtraction

$h(x) = (5x^2 + 2x) - (-3x^2)$

$\boxed{h(x) = 8x^2 + 2x}$

c. Find $h(x) = f(x) \cdot g(x)$.

Operation: multiplication

$h(x) = (5x^2 + 2x)(-3x^2)$

$\boxed{h(x) = -15x^4 - 6x^3}$

d. Find $h(x) = \dfrac{f(x)}{g(x)}$.

Operation: Division

$h(x) = \dfrac{5x^2 + 2x}{-3x^2} = \dfrac{\cancel{x}(5x+2)}{\cancel{x}(-3x)}$

$\boxed{h(x) = \dfrac{5x+2}{-3x}}$

When finding the domain algebraically, there are two "problem areas" we look for in the equation. (If there are no "problem areas", the domain is $(-\infty, \infty)$ (or "all real numbers").

1. variable under an even radical. (Anything under an even radical has to be positive.) $\sqrt{x}$: x has to be positive.

2. variable in the denominator. (Anything in the denominator CANNOT be zero.)

$\frac{1}{x}$: $x \neq 0$

Find the domain (algebraically) of the following functions.

a. $f(x) = \sqrt{2x - 8}$

$2x - 8 \geq 0$
$+8 \quad +8$
$\cancel{2}x \geq \frac{8}{2}$

$x \geq 4$

4

$\boxed{D: [4, \infty)}$

b. $u(x) = \dfrac{x - 5}{2x + 4}$

$2x + 4 \neq 0$
$-4 \quad -4$
$\cancel{2}x \neq \frac{-4}{2}$

$x \neq -2$

-2

$\boxed{D: (-\infty, -2) \cup (-2, \infty)}$

c. $m(x) = \sqrt{9 - 3x}$

$9 - 3x \geq 0$
$-9 \quad -9$
$\cancel{-3}x \geq \frac{-9}{-3}$

$x \leq 3$

3

$\boxed{D: (-\infty, 3]}$

d. $d(x) = x + 3$

$\boxed{D: (-\infty, \infty)}$

D: all real numbers

D: $\mathbb{R}$

e. $\dfrac{x+3}{\sqrt{x+1}}$

$x + 1 > 0$
$-1 \quad -1$
$x > -1$

-1

$\boxed{D: (-1, \infty)}$

Example 2: Let $f(x) = 2x + 1$ and $g(x) = x^2 + 3x - 4$. Perform the indicated operation and state the domain of the new function.

D: $(-\infty, \infty)$ D: $(-\infty, \infty)$

a. Find $h(x) = f(x) + g(x)$.

Operation: addition

$h(x) = (2x+1) + (x^2 + 3x - 4)$

$\boxed{h(x) = x^2 + 5x - 3}$

Domain: $(-\infty, \infty)$

b. Find $h(x) = f(x) - g(x)$.

Operation: subtraction

$h(x) = (2x+1) - (x^2 + 3x - 4)$

$\boxed{h(x) = -x^2 - x + 5}$

Domain: $(-\infty, \infty)$

c. Find $h(x) = f(x) \cdot g(x)$.

Operation: multiplication

$h(x) = (2x+1)(x^2 + 3x - 4)$

$h(x) = 2x^3 + 6x^2 - 8x + x^2 + 3x - 4$

$\boxed{h(x) = 2x^3 + 7x^2 - 5x - 4}$

Domain: $(-\infty, \infty)$

d. Find $h(x) = 2f(x) - g(x)$.

Operation: multiplication & subtraction

$h(x) = 2(2x+1) - (x^2 + 3x - 4)$

$h(x) = 4x + 2 - (x^2 + 3x - 4)$

$\boxed{h(x) = -x^2 + x + 6}$

Domain: $(-\infty, \infty)$

e. Find $h(x) = \dfrac{f(x)}{g(x)}$.

Operation: division

$\boxed{h(x) = \dfrac{2x+1}{x^2 + 3x - 4}} = \dfrac{2x+1}{(x+4)(x-1)}$

$(x+4)(x-1) \neq 0$

$x + 4 \neq 0$ $x - 1 \neq 0$

$-4 \;\; -4$

$x \neq -4$ $x \neq 1$

Domain: $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

f. Find $h(x) = \dfrac{g(x)}{f(x)}$.

Operation: Division

$\boxed{h(x) = \dfrac{x^2 + 3x - 4}{2x+1}} = \boxed{\dfrac{(x+4)(x-1)}{2x+1}}$

$2x + 1 \neq 0$

$-1 \;\; -1$

$\dfrac{2x}{2} \neq \dfrac{-1}{2}$ $x \neq \dfrac{-1}{2}$

Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

g. Find $h(x) = \dfrac{2f(x)}{f(x)}$.

Operation: multiplication & division

$h(x) = \dfrac{2\cancel{(2x+1)}}{\cancel{(2x+1)}} = 2$

$\boxed{h(x) = 2}$

$2x + 1 \neq 0$ $x \neq \frac{-1}{2}$

$-1 \;\; -1$

$2x \neq -1$

Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

HW: Pg. 271: 1-23
you MUST do on a seperate piece of paper

h. Find $h(x) = f(4) - g(-2)$.

Operation:

$h(x) = f(4) - g(-2)$

$f(4) = 2(4) + 1$

$= 8 + 1 = 9$

$g(-2) = (-2)^2 + 3(-2) - 4$

$= 4 - 6 - 4$

$= -6$

$h(x) = 9 - (-6) = 15$

$\boxed{h(x) = 15}$