NOTES: SECONDARY 2 HONORS

INVERSES OF FUNCTIONS (3.5)

Vocabulary

- **Inverse relation:**

Example: Identify the domain and range of the original relation. Find the inverse relation and identify the domain and range of the inverse relation. Is the inverse relation a function?

a. $\{(1,-1),(2,-2),(-3,3),(4,-4),(5,-5)\}$

D: $\{-3,1,2,4,5\}$ R: $\{-5,-4,-2,-1,3\}$

inverse: $\{(-1,1),(-2,2),(3,-3),(-4,4),(-5,5)\}$

D: $\{-5,-4,-2,-1,3\}$ R: $\{-3,1,2,4,5\}$

yes, the inverse is a function.

b. $\{(-4,2),(-2,1),(0,0),(2,1),(4,2)\}$

D: $\{-4,-2,0,2,4\}$ R: $\{0,1,2\}$

inverse: $\{(2,-4),(1,-2),(0,0),(1,2),(2,4)\}$

D: $\{0,1,2\}$ R: $\{-4,-2,0,2,4\}$

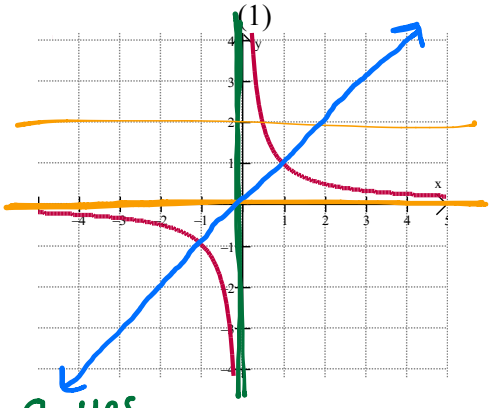
No, the inverse is not a function

Example: Answer the following questions about each graph below. Explain your answer in complete sentences. Then graph the inverse of each relation.

a. Is the relation a function? (Vertical Line Test)

b. Is the relation's inverse a function? (Horizontal Line Test)

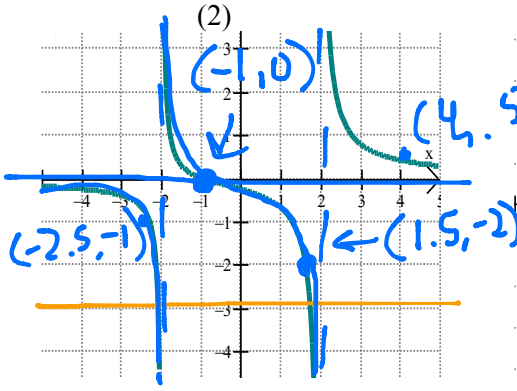
(1)



a. yes

b. yes

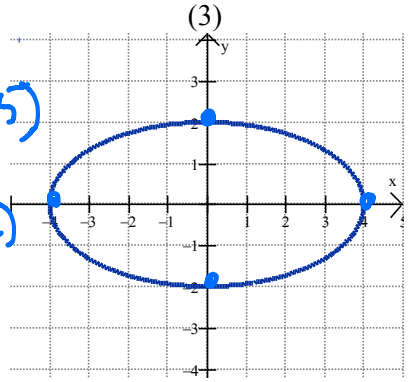
(2)



a. yes

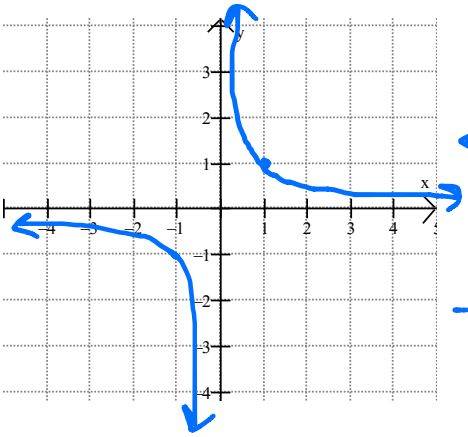
b. no

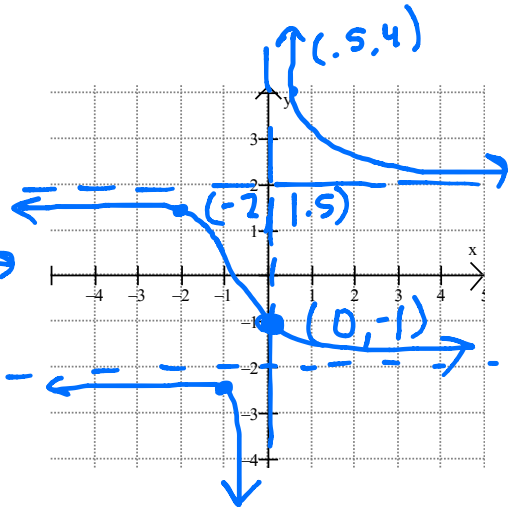
(3)

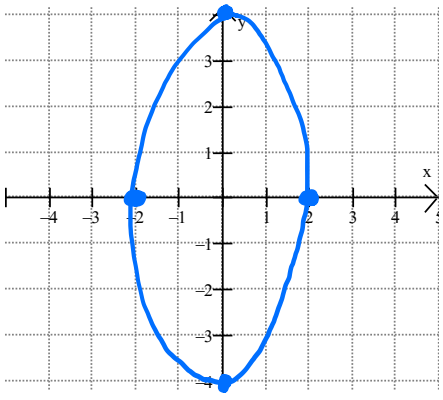


a. no

b. no







Finding an inverse algebraically:

- switch "x" and "y"
- solve for y.

Example: Find the inverse of the following algebraically.

1. $f(x) = 3x + 2$

$x = 3y + 2$
$-2 \quad -2$
$\frac{x-2}{3} = \frac{3y}{3}$
$f^{-1}(x) = \frac{x-2}{3}$

2. $f(x) = \frac{-4}{5}x + 1$

$x = \frac{-4}{5}y + 1$
$-1 \quad -1$
$-\frac{5}{4}(x-1) = (-\frac{4}{5}y)(-\frac{5}{4})$
$f^{-1}(x) = \frac{-5(x-1)}{4}$

3. $f(x) = 3x^2 - 5$

$x = 3y^2 - 5$
$+5 \quad +5$
$\frac{x+5}{3} = \frac{3y^2}{3}$
$\sqrt{y^2} = \sqrt{\frac{x+5}{3}}$
$f^{-1}(x) = \pm\sqrt{\frac{x+5}{3}}$

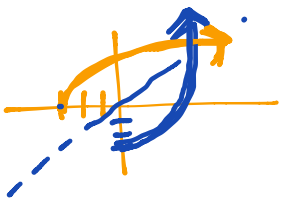
4. $f(x) = \frac{x+5}{x-1}$

$(y-1)x = \frac{y+5}{y-1}(y-1)$
$x(y-1) = y + 5$
$xy - x = y + 5$
$-y \quad -y$
$xy - y - x = 5$
$+x \quad +x$
$xy - y = x + 5$
$y(x-1) = x + 5$
$\frac{x-1}{x-1} \quad \frac{x-1}{x-1}$
$f^{-1}(x) = \frac{x+5}{x-1}$

Be aware of the restricted domain of inverses.

Find the domain of $f(x) = \sqrt{x+3}$.

$x + 3 \geq 0$
$x \geq -3$
$[-3, \infty)$



Find $f^{-1}(x)$.

$x = \sqrt{y+3}$
$x^2 = y + 3$
$y = x^2 - 3$

Identify the domain of $f^{-1}(x)$.

$[0, \infty)$

Vocabulary

- **Even function:**

Symmetric about the y-axis.

$f(-x) = f(x)$

- **Odd function:**

Symmetric about the origin

$f(-x) = -f(x)$

Example: Determine algebraically if the function is even, odd, or neither even nor odd.

1. $f(x) = |x| - 1$

$f(-x) = |-x| - 1$
$= |x| - 1$

EVEN

2. $f(x) = -3x^4 - 4x^2$

$f(-x) = -3(-x)^4 - 4(-x)^2$
$= -3x^4 - 4x^2$

EVEN

3. $f(x) = x^2 - 4x + 4$

$f(-x) = (-x)^2 - 4(-x) + 4$
$= x^2 + 4x + 4$

Neither

4. ~~$y = |x| + 4$~~ $y = x^5 - 3x^3 + x$

$f(-x) = (-x)^5 - 3(-x)^3 + (-x)$
$= -x^5 + 3x^3 - x$

ODD