NOTES: SECONDARY 2 HONORS

APPLICATION OF QUADRATIC EQUATIONS (4.6)

STARTER

1. Factor completely $f(x) = x^3 + x^2 - 12x$

$f(x) = x(x^2 + x - 12)$

$\boxed{f(x) = x(x+4)(x-3)}$

2. Solve by factoring $0 = 5x^2 - 9x + 4$

$0 = (5x-4)(x-1)$

$5x - 4 = 0 \qquad x - 1 = 0$

$\boxed{x = \frac{4}{5}} \qquad \boxed{x = 1}$

3. Find the zeros $f(x) = 5x^2 + 16x - 16$

$0 = 5x^2 + 16x - 16$

$= 5x^2 + 20x - 4x - 16$

$= 5x(x+4) - 4(x+4)$

$= (x+4)(5x-4)$

$\boxed{x = -4} \quad \boxed{x = \frac{4}{5}}$

4. Find the vertex. $f(x) = x^2 + 14x + 24$

$f(x) = (x^2 + 14x + \underline{49}) + 24 - \underline{49}$

$f(x) = (x+7)^2 - 25$

$\boxed{\text{vertex: } (-7, -25)}$

When solving contextual type problems, it is important to:

- Identify what you know.
- Determine what you are trying to find and define your variables.
- Draw a picture to help you visualize the situation when possible. Remember to label all parts of your drawing.
- Use familiar formulas to help you write equations.
- Check your answer for reasonableness and accuracy.
- Make sure you answered the entire question.
- Use appropriate units.

Example: Find three consecutive integers such that the product of the first two plus the square of the third is equal to 137.

1st number: x
2nd number: $x+1$
3rd number: $x+2$

$x(x+1) + (x+2)^2 = 137$

$(x+2)(x+2)$

$x^2 + x + x^2 + 4x + 4 - 137 = 0$

$2x^2 + 5x - 133 = 0$

$2x^2 + 19x - 14x - 133 = 0$

$x(2x+19) - 7(2x+19) = 0$

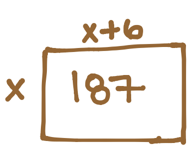
$(2x+19)(x-7) = 0$

$2x + 19 = 0 \qquad x - 7 = 0$

$x = \cancel{-\frac{19}{2}} \qquad x = 7$

The three numbers are 7, 8 and 9.

Example: A photo is 6 in. longer than it is wide. Find the length and width of the photo if the area is 187 in^2.



$x(x+6) = 187$

$x^2 + 6x = 187$

$x^2 + 6x - 187 = 0$

$(x-11)(x+17) = 0$

$x = 11 \quad x = \cancel{-17}$

The width of the photo is 11 inches and the length of the photo is 17 inches.

Example: Find two numbers that add to 150 and have a maximum product. What is the maximum product?

$x + y = 150 \qquad x = -y + 150$

$M.P. = xy$

$M.P. = y(-y + 150)$

$M.P. = -y^2 + 150y$

$M.P. = -(y^2 - 150y + \underline{5625}) - (-1)\underline{5625}$

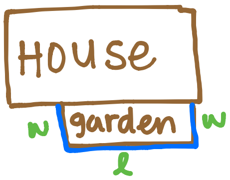
$M.P. = -(y^2 - 150y + 5625) + 5625$

$M.P. = -(y - 75)^2 + 5625$

vertex: $(75, 5625)$

The maximum product is 5625 when the two numbers are 75 and 75.

Example: Jason wants to fence in a rectangular garden in his backyard. If one side of the garden is against the house and Jason has 48 feet of fencing, what dimensions will maximize the garden area while utilizing all of the fencing?



$2w + l = 48 \rightarrow l = -2w + 48$

$A = wl$

$A = w(-2w + 48)$

$A = -2w^2 + 48w$

$A = -2(w^2 - 24w + \underline{144}) - (-2)\underline{144}$

$\frac{-24}{2} = -12 \quad (-12)^2 = 144$

$A = -2(w^2 - 24w + 144) + 288$

$A = -2(w - 12)^2 + 288$

vertex: $(\underline{\underline{12}}, 288)$

The width of the garden needs to be 12 feet and the length needs to be 24 feet for a maximum area of $288 ft^2$.

Example: The Willis Tower (formerly Sears Tower) in Chicago, Illinois is the tallest building in the United States. It is 108 stories or about 1,451 feet high. (Assume that each floor is 13 feet high.)

a. A window washer is 28 floors from the top and he drops a piece of equipment, how long will it take for the equipment to reach the ground? (Use the equation $h(t) = -16t^2 + v_0 t + h_0$ where v_0 represents the initial velocity and h_0 represents the initial height.

$h(t) = 0$

$v_0 = 0$

$h_0 = (108 - 28)(13)$
$= 1040$

$h(t) = -16t^2 + 1040$

$0 = -16t^2 + 1040$

$\frac{-1040}{-16} = \frac{-16t^2}{-16}$

$\sqrt{t^2} = \sqrt{65}$

$t = \pm\sqrt{65}$

The equipment will reach the ground at $\sqrt{65}$ seconds.

b. How far from the ground is the piece of equipment after 5 seconds?

$h(5) = -16(5)^2 + 1040$
$= -400 + 1040$
$= 640$

The equipment is 640 feet from the ground at 5 seconds.

c. When does the equipment pass the 16th floor?

$16(13) = 208$

The 16th floor is 208 ft. off the ground.

$208 = -16t^2 + 1040$

$\frac{-832}{-16} = \frac{-16t^2}{-16}$

$52 = t^2$

$t = \pm 2\sqrt{13}$

The equipment will pass the 16th floor at $2\sqrt{13}$ seconds.

Example: The Salt Lake Bees are planning to have a fireworks display after their game with the Tacoma Rainiers. Their launch platform is 5 feet off the ground and the fireworks will be launched with an initial velocity of 32 feet per second. How long will it take each firework to reach their maximum height?

$h(t) = -16t^2 + v_0 t + h_0$

$v_0 = 32$

$h_0 = 5$

$h(t) = -16t^2 + 32t + 5$

$h(t) = (-16t^2 + 32t + \_\_) + 5 - \_\_$

$= -16(t^2 - 2t + \underline{1}) + 5 - (-16)\underline{1}$

$= -16(t - 1)^2 + 21$

vertex: $(1, 21)$

The firework will reach its maximum height of 21 feet at 1 second.