NOTES: SECONDARY 2 HONORS

COMPLETE THE SQUARE (4.4)

STARTER

$y=a(x-h)^2+k$ vertex: (h,k)

1. Find the x-intercepts of $f(x)=2x^2-9x+4$.

$0=2x^2-9x+4$

$0=(x-4)(2x-1)$

$x-4=0$ $2x-1=0$

$x=4$ $x=\frac{1}{2}$

$\boxed{(4,0)\ \&\ (\tfrac{1}{2},0)}$

2. Determine the vertex of $f(x)=-2(x+3)^2-1$

vertex: $(-3,-1)$

3. Identify the max/min value of $f(x)=-2(x+3)^2-1$

max value at -1 when x is -3

4. Find the zeros of $f(x)=2(x+3)^2-36$

$0=2(x+3)^2-36$

$+36 \quad +36$

$\frac{36}{2}=\frac{\cancel{2}(x+3)^2}{\cancel{2}}$

$\sqrt{18}=\sqrt{(x+3)^2}$

$x+3=\pm\sqrt{18}$

$x+3=\pm 3\sqrt{2}$

$-3 \quad -3$

$\boxed{x=-3\pm 3\sqrt{2}}$

5. Factor $4x^2-36x+81$

$2(2)(9)=36$

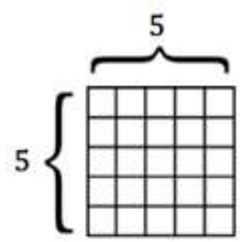
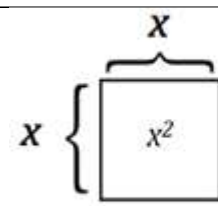
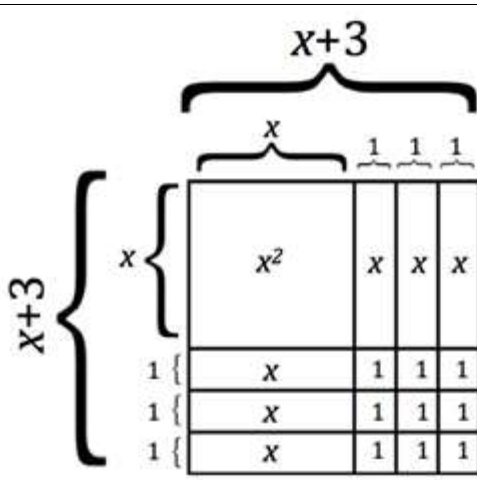
$\boxed{(2x-9)^2}$

6. Factor $ax^2+2abx+b^2$

$2(a)(b)=2ab$

$\boxed{(ax+b)^2}$

PERFECT SQUARES

$5^2=25$	
x^2	
$(x+3)^2=x^2+6x+9$	

RECALL:

Standard Form: $ax^2 + bx + c$

Vertex Form: $a(x-h)^2 + k$ where $(x-h)^2$ represents the square__________ portion of the quadratic expression.

QUESTION: What does complete the square give us? the vertex and max/min

STEPS TO COMPLETE THE SQUARE As we go through the steps, look at the equation $2x^2 + 4x = 9$	
Step 1: Set the equation equal to zero. $ax^2 + bx + c = 0$	$2x^2 + 4x = 9$ $-9 \quad -9$ $2x^2 + 4x - 9 = 0$
Step 2: Separate the x^2 and the x terms from the "c" term. Leave blanks for the missing values. $(ax^2 + bx + \_\_\_\_\_) + c - \_\_\_\_\_ = 0$	$(2x^2 + 4x + \_\_) - 9 - \_\_ = 0$
Step 3: Factor out the "a" (or the number in front of x^2). Don't forget to divide/multiply the missing value next to "c" by "a". We need to keep the expression equivalent. $a\left(x^2 + \frac{b}{a}x + \_\_\_\_\_\right) + c - (a)\_\_\_\_\_ = 0$	$2(x^2 + 2x + \_\_) - 9 - (2)\_\_ = 0$
Step 4: Take your new "b" (or the number in front of "x") and divide it by 2.	$\frac{2}{2} = 1$
Step 5: Take the result from step #4 and square it.	$(1)^2 = 1$
Step 6: Add the result from step 5 to both missing values. (Remember you are adding to one side and subtracting from the other.)	$2(x^2 + 2x + 1) - 9 - (2)1 = 0$ $2(x^2 + 2x + 1) - 11 = 0$
Step 7: Factor the perfect square trinomial. This should always factor into $(x + your\ answer\ from\ step\ 4)^2$	$2(x+1)^2 - 11 = 0$
Step 8: Simplify as much as you can.	$2(x+1)^2 - 11 = 0$ * solve for x if needed.

Example: For the following functions

a. Rewrite the function in vertex form of $f(x) = a(x-h)^2 + k$ complete the square.

b. Determine the vertex.

c. Determine if the vertex is a maximum or a minimum and state the maximum or minimum value.

d. Determine the y-intercept. (Remember y-intercept is a point)

e. Determine the x-intercept(s). (Remember x-intercept is a point)

f. Sketch the graph of the function with its axis of symmetry (label the vertex and intercepts on the graph).

1. $f(x) = x^2 - 4x - 32$

$f(x) = (x^2 - 4x + \_\_) - 32 - \_\_$

$\frac{-4}{2} = -2 \quad (-2)^2 = 4$

$f(x) = (x^2 - 4x + 4) - 32 - 4$

$f(x) = (x^2 - 4x + 4) - 36$

a. $f(x) = (x-2)^2 - 36$

b. vertex: $(2, -36)$

c. min value at -36 when x is 2.

d. $f(0) = 0^2 - 4(0) - 32 = -32 \quad (0, -32)$

e. $0 = (x-2)^2 - 36$ $\quad (8,0)$ & $(-4,0)$

$+36 \qquad +36$

$36 = (x-2)^2$

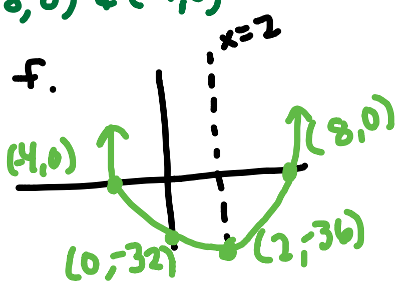
$\pm\sqrt{36} = x-2$

$\pm 6 = x - 2$

$x - 2 = 6 \qquad x - 2 = -6$

$x = 8 \qquad x = -4$

f.



2. $f(x) = x^2 + 6x + 7$

3. $f(x) = x^2 - 5x - 7$

$f(x) = (x^2 - 5x + \frac{25}{4}) - 7 - \frac{25}{4}$

$\frac{-5}{2} \quad (\frac{-5}{2})^2 = \frac{25}{4}$

$f(x) = (x - \frac{5}{2})^2 - \frac{53}{4}$

a. $f(x) = (x - 5/2)^2 - \frac{53}{4}$

b. vertex: $(\frac{5}{2}, -\frac{53}{4})$

c. Min value at $-53/4$ when $x = 5/2$

d. $f(0) = 0^2 - 5(0) - 7 = -7 \quad \boxed{(0, -7)}$

e. $0 = (x - 5/2)^2 - \frac{53}{4}$

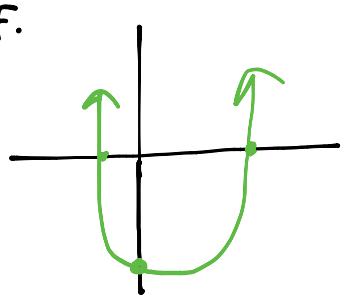
$+\frac{53}{4} \qquad +\frac{53}{4}$

$\sqrt{\frac{53}{4}} = \sqrt{(x - \frac{5}{2})^2}$

$\frac{\pm\sqrt{53}}{2} = x - \frac{5}{2} \quad \boxed{x = \frac{5}{2} \pm \frac{\sqrt{53}}{2}}$

$x \approx -1.14 \quad x \approx 6.14$

f.



4. $f(x) = -3x^2 + 6x + 9$

$f(x) = (-3x^2 + 6x + \_\_) + 9 - \_\_$

$f(x) = -3(x^2 - 2x + \_\_) + 9 - (-3)\_\_$

$\frac{-2}{2} = -1 \quad (-1)^2 = 1$

$f(x) = -3(x^2 - 2x + 1) + 9 - (-3)(1)$

$f(x) = -3(x-1)^2 + 9 + 3$

a. $f(x) = -3(x-1)^2 + 12$

b. vertex: $(1, 12)$

c. max value at 12 when x is 1.

d. $f(0) = 9 \quad (0, 9)$

e. $0 = -3(x-1)^2 + 12$

$-12 \qquad -12$

$\frac{-12}{-3} = \frac{-3(x-1)^2}{-3}$

$4 = (x-1)^2$

$\pm\sqrt{4} = x - 1$

$\pm 2 = x - 1$

$x - 1 = 2 \qquad x - 1 = -2$

$x = 3 \qquad x = -1$

$(3, 0) \quad (-1, 0)$

When do you factor and when do you complete the square?

Example: The height $h(t)$, in feet, of a "weeping willow" firework display, t seconds after having been launched from a $80\,ft$ high rooftop, is given by $h(t) = -16t^2 + 64t + 80$.

a. Find the zeros of the function and explain the meaning in the context of the problem.

Factor

$0 = -16t^2 + 64t + 80$

$0 = -16(t^2 - 4t - 5)$

$0 = -16(t-5)(t+1)$

$t = 5 \quad \cancel{t = -1}$

$t = 5$

The fireworks hit the ground at 5 seconds.

b. Find the vertex of the function and explain the meaning in the context of the problem.

Vertex Form: complete the square.

$f(x) = (-16t^2 + 64t + \_\_\_) + 80 - \_\_\_$

$= -16(t^2 - 4t + \underline{4}) + 80 - \underline{(-16)(4)}$

$= -16(t-2)^2 + 80 + 64$

$= -16(t-2)^2 + 144$

vertex: $(2, 144)$

At 2 seconds, the firework has reached its max height of 144 ft.

Example: The value of some stock can by represented by $V(x) = 2x^2 - 8x + 10$, in thousands of dollars, where x is the number of months after January 2013. Find the vertex of the function and explain the meaning in the context of the problem.

Complete the square

$V(x) = 2x^2 - 8x + 10$

$= (2x^2 - 8x + \_\_\_) + 10 - \_\_\_$

$= 2(x^2 - 4x + \underline{4}) + 10 - \underline{(2)(4)}$

$= 2(x-2)^2 + 2$

vertex: $(2, 2)$

The value of the stock is at its min value of \$2000 in March 2013.

Example: A projectile is thrown upward so that its distance above the ground after t seconds is $h(t) = -12t^2 + 540t$. When does it reach its maximum height? When does it hit the ground?

↓
vertex