

NOTES: SECONDARY 2 HONORS
FACTOR A QUADRATIC EXPRESSION OF THE FORM $ax^2 + bx + c$ (3.2)

STARTER

Factor by find the GCF.

1. $3v + 15$

$3(v + 5)$

2. $4y^2 - 12y$

$4y(y - 3)$

3. $-7x - 4$

$-(7x + 4)$

4. $-6m^2 + 20m$

$-2m(3m - 10)$

5. $x^2 + 16x - 17$

$(x - 1)(x + 17)$

6. $y^2 + 7y + 12$

$(y + 3)(y + 4)$

7. $x^2 - 14x - 15$

$(x - 15)(x + 1)$

Vocabulary

- Polynomial:
An expression with multiple terms.
- Monomial:
An expression with one term
- Binomial:
An expression with two terms
- Trinomial:
An expression with three terms.

FACTOR BY GROUPING

You want to factor by grouping when there are 4 or more terms. (REMEMBER to have to the polynomial in standard form)

Example: Factor $5x^2 + x + 10x + 2$

$x(5x + 1) + 2(5x + 1)$

$(5x + 1)(x + 2)$

Practice Exercises

1. $y(y + 3) + 4(y + 3) = (y + 3)(y + 4)$

2. $5u^2 + u + 10u + 2$

3.

$2x^2 + x - 6x - 3$
 $x(2x + 1) - 3(2x + 1)$

$= (2x + 1)(x - 3)$

$$ax^2 + bx + c$$

$$a \neq 1$$

FACTOR BY USING THE "AC" METHOD

Factor $7x^2 + 15x + 2$	
$a = 7$ $c = 2$ $ac = 7(2) = 14$	Multiply the leading coefficient and the constant (the "a" term and the "c" terms)
15	Identify the coefficient of the middle term (the "b" term)
$\begin{array}{r l} 14 & 15 \\ \hline 1, 14 & 15 \end{array}$ $1 \text{ and } 14$	Find two numbers that multiply to the leading coefficient and the constant term ("ac") but add to the coefficient of the middle term. ("b")
$7x^2 + 15x + 2$ $7x^2 + x + 14x + 2$	Rewrite the original polynomial by replacing the middle term ("b") with the two numbers found above. Your statement must be <i>equivalent</i> .
$\underline{7x^2 + x} + \underline{14x + 2}$	Group the first two terms and the second two terms
$x(7x+1) + 2(7x+1)$	Factor out the Greatest Common Factor from each group.
$(7x+1)(x+2)$	Are the parentheses identical? If so, factor out the terms in the parenthesis. Write the remaining factor together in parenthesis to complete your answer.

Examples: Factor by using the "ac" method.

1. $3x^2 - 4x - 4$

$$\begin{array}{r|l} -12 & -4 \\ \hline -6, 2 & -4 \end{array}$$

$$3x^2 - 6x + 2x - 4$$

$$3x(x-2) + 2(x-2)$$

$$= (x-2)(3x+2)$$

2. $2x^2 + 5x + 3$

$$\begin{array}{r|l} 6 & 3 \\ \hline 3, 2 & 3 \end{array}$$

$$2x^2 + 3x + 2x + 3$$

$$x(2x+3) + 1(2x+3)$$

$$= (2x+3)(x+1)$$

3. $5y^2 - 3y - 2$

$$\begin{array}{r|l} -10 & -3 \\ \hline -5, 2 & -3 \end{array}$$

$$5y^2 - 5y + 2y - 2$$

$$5y(y-1) + 2(y-1)$$

$$= (y-1)(5y+2)$$

FACTOR BY USING THE "GUESS AND CHECK" METHOD

Factor $6x^2 + 19x + 3$			
Factors of $6x^2$	Factors of 3	Possible Binomial Factors	Sum of Outer and Inner Products
$3x, 2x$ $(3x \quad)(2x \quad)$ <small>or</small> $(2x \quad)(3x \quad)$ <small>or</small>	$3, 1$ $(\quad 3)(\quad 1)$ <small>or</small> $(\quad 1)(\quad 3)$ <small>or</small>	$(3x + 3)(2x + 1)$ $(3x + 1)(2x + 3)$	$3x + 6x = 9x$ $9x + 2x = 11x$
$6x, x$ $(6x \quad)(x \quad)$ <small>or</small> $(6x \quad)(x \quad)$	$3, 1$ $(\quad 3)(\quad 1)$ <small>or</small> $(\quad 1)(\quad 3)$	$(6x + 3)(x + 1)$ $(6x + 1)(x + 3)$	$6x + 3x = 9x$ $18x + x = \boxed{19x}$ <i>desired middle term.</i>

So, $6x^2 + 19x + 3 = (6x + 1)(x + 3)$. Check by multiplying.

Examples: Factor by using the "guess and check" method.

1. $3x^2 - 4x - 7$

$$(3x - 7)(x + 1)$$

2. $4y^2 - y - 5$

$$(4y - 5)(y + 1)$$

3. $5b^2 + b - 4$

$$(5b - 4)(b + 1)$$

Factor using whatever method you choose. Some problems are *not* factorable. Write prime if problems are not factorable.

1. $2j^2 + 9j + 9$

$$\begin{array}{r} 18 \ 9 \\ 6 \ 3 \overline{) 9} \end{array} \quad \begin{array}{l} 2j^2 + 6j + 3j + 9 \\ \underline{2j(j+3) + 3(j+3)} \\ \hline (j+3)(2j+3) \end{array}$$

2. $7x^2 + 2x - 2$

$$\frac{-14 \ 2}{1} \quad \boxed{\text{PRIME}}$$

3. $4w^2 - 4w - 3$

$$(2w + 1)(2w - 3)$$

4. $8k^2 + 14k + 5$

$$(2k + 1)(4k + 5)$$

5. $2d^2 - 7df + 6f^2$

$$(2d - 3f)(d - 2f)$$

6. $5x^2 - 13x - 6$

$$(5x + 2)(x - 3)$$

7. $2m^2 + 7m + 5$

$$(2m + 5)(m + 1)$$

8. $2x^2 + 9x - 11$

$$(2x + 11)(x - 1)$$

9. $3x^2 - 22x - 1$

Prime

10. $3x^2 - x - 5$

Prime

11. $4y^2 - 16y - 9$

$$(2y - 9)(2y + 1)$$

12. $20z^2 + 49z + 30$

$$(4z + 5)(5z + 6)$$

13. $26p^2 + 29p - 15$

$$(2p + 3)(13p - 5)$$

14. $3a^2 - ab - 2b^2$

$$(3a + 2b)(a - b)$$

15. $4x^2 - 18x + 9$

Prime

FACTOR A PERFECT SQUARE TRINOMIAL

A perfect square trinomial is the result when you square a binomial.

$$(x + 4)^2 = (x + 4)(x + 4) = \underline{x^2} + \underline{8x} + \underline{16}$$

The square of
the first term
Twice the
product of the
two terms
The square of
the last term

Identifying a perfect square trinomial can make factoring easier, since all perfect square trinomials follow the same pattern, which is:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Again, notice that the first and last terms of the trinomial are always the square of a and square of b (first and last terms of the factors). The middle term is always twice the product of the two terms or $2ab$. If the polynomial doesn't follow this pattern, it is not a perfect square trinomial.

Example: Is the polynomial a perfect square trinomial? Explain your reasoning.

1. $x^2 + 14x + 49$
 $2(1)(7) = 14$
 yes
 $(x+7)^2$

2. $4x^2 - 12x + 9$
 $2(2)(3) = 12$
 yes
 $(2x-3)^2$

3. $9x^2 + 12x + 16$
 No

Example: Factor. If not factorable, write *prime*.

1. $x^2 + 12x + 144$
 prime

2. $9x^2 - 30x + 25$
 $(3x-5)^2$

3. $x^2 + 4x - 4$
 $2(1)(2) = 4$

4. $64p^2 - 16p + 1$
 $(8p-1)^2$

5. $81x^2 - 36x + 4$
 $2(9)(2) = 36$
 yes
 $(9x-2)^2$

6. $4x^2 - 18x + 9$
 prime

7. $x^2 + 2x + 1$
 $(x+1)^2$

8. $4x^2 - 18x + 9$
 prime

9. $9x^2 - 24xy + 16y^2$
 $(3x-4y)^2$

Explain why the last term of every perfect square trinomial is a positive value.

$$25v^2 - 16$$

$$= (5v + 4)(5v - 4)$$