UNIT 4 STUDY GUIDE

1. a. $y = 2(x+3)^2 - 8$

$y = 2(x+3)(x+3) - 8 = 2(x^2+6x+9) - 8 = 2x^2+12x+18-8$

$\boxed{y = 2x^2+12x+10}$

b. $y = (2x-3)(3x-1)$

$y = 6x^2 - 2x - 9x + 3$ $\quad \boxed{y = 6x^2 - 11x + 3}$

2. a. $y = 3x^2 + 13x - 30$ * Intercept form is the same as factored form.

$\boxed{y = (3x-5)(x+6)}$

b. $y = (2x+3)(2x-5)$ * This function is already in intercept form.

$\boxed{y = (2x+3)(2x-5)}$

3. a. $f(x) = 3x^2 + 18x + 15$ * To write a function in vertex form: COMPLETE THE SQUARE.

$f(x) = (3x^2 + 18x + \_\_\_) + 15 - \_\_\_$

$f(x) = 3(x^2 + 6x + \_\_\_) + 15 - (3)\_\_\_$ $\quad \frac{6}{2} = 3 \quad (3)^2 = 9$

$f(x) = 3(x^2 + 6x + \underline{9}) + 15 - 3(\underline{9})$

$f(x) = 3(x+3)^2 + 15 - 27$

$\boxed{f(x) = 3(x+3)^2 - 12}$

b. $f(x) = 6(x-3)(x+1)$

$f(x) = 6(x^2 - 2x - 3) = 6x^2 - 12x - 18$

$f(x) = 6(x^2 - 2x + \_\_\_) - 18 - (6)\_\_\_$ $\quad \frac{-2}{2} = -1 \quad (-1)^2 = 1$

$f(x) = 6(x^2 - 2x + 1) - 18 - 6(1)$

$f(x) = 6(x-1)^2 - 18 - 6$

$\boxed{f(x) = 6(x-1)^2 - 24}$

4. a. $5k^3 + 13k^2 - 6k$ * Look for a GCF first. Factor using "ac" method or "guess or check".

$k(5k^2 + 13k - 6)$

$k(\underbrace{5k^2 + 15k}\,\underbrace{-2k - 6})$

$k(5k(k+3) - 2(k+3)) = \boxed{k(k+3)(5k-2)}$

b. $10m^2 + 56m - 24$

$2(5m^2 + 28m - 12) = 2(5m^2 + 30m - 2m - 12)$

$= 2(5m(m+6) - 2(m+6)) = \boxed{2(m+6)(5m-2)}$

c. $7a^2 + 27ab + 18b^2$

$7a^2 + 21ab + 6ab + 18b^2$

$7a(a+3b) + 6b(a+3b) = \boxed{(a+3b)(7a+6b)}$

d. $x^2 + 20x + 100$ *This is a perfect square trinomial.

$\boxed{(x+10)^2}$

5. $f(x) = 2x^2 - 9x - 56$

a. Find the x-intercepts. To find the x-intercepts, solve by factoring. (Remember: $y=0$)

$0 = 2x^2 - 9x - 56$

$0 = (2x+7)(x-8)$

$2x + 7 = 0$ $\quad$ $x - 8 = 0$

$x = \frac{-7}{2}$ $\quad$ $x = 8$

$\boxed{(\frac{-7}{2}, 0)}$ & $\boxed{(8,0)}$

b. Find the y-intercept. $x = 0$

$f(0) = 2(0)^2 - 9(0) - 56$

$= 0 - 56$

$= -56$

$\boxed{(0, -56)}$

6. $4(x-3)^2 = 80$

$\sqrt{(x-3)^2} = \sqrt{20}$

$x - 3 = \pm 2\sqrt{5}$

$+3 \quad +3$

$\boxed{x = 3 \pm 2\sqrt{5}}$

7. $x = \frac{-5}{2}$ $\quad$ $x = 3$

$(x + \frac{5}{2})(x-3) = (2x+5)(x-3)$ FOIL

$= \boxed{2x^2 - x - 6}$

8. $R(p) = -4p^2 + 1280p$, p = unit price in dollars; $R(p)$ = profit

To find the maximum, find the vertex. To find the vertex, put $R(p)$ in vertex form. To put a function in vertex form, complete the square.

$R(p) = -4(p^2 - 320p) - (-4)$___ $\qquad \frac{-320}{2} = -160 \qquad (-160)^2 = 25{,}600$

$= -4(p^2 - 320p + 25600) - (-4)(25{,}600)$

$= -4(p-160)^2 + 102{,}400$

vertex: $(160, 102{,}400)$.

If the manufacturer sets the price p at \$160, the revenue would be maximized at \$102,400.

9. $h(t) = -11t^2 + 286t$

a. Find the zeros. To find the zeros, set $y = 0$ and solve (FACTOR)

$0 = -11t^2 + 286t$

$0 = -11t(t - 26)$

$0 = -11t \qquad 0 = t - 26$

$t = 0 \qquad t = 26$

$(0,0)$ and $(26, 0)$

The projectile is launched from the ground $(0,0)$ and hits the ground again after 26 seconds.

b. Find the vertex. complete the square.

$h(t) = -11(t^2 - 26t + \_\_) - (-11)\_\_ \qquad \frac{-26}{2} = -13 \qquad (-13)^2 = 169$

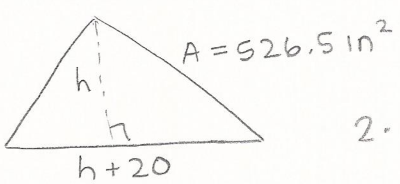
$= -11(t^2 - 26t + 169) - (-11)(169)$

$= -11(t-13)^2 + 1859$.

vertex: $(13, 1859)$

The projectile reaches a max height of 1859 units after 13 seconds.

10. Triangle-topped table, 20 inches longer (base) than the height. Area $= 526.5\,in^2$



Area of a triangle $= \frac{b \cdot h}{2}$

$2 \cdot 526.5 = \frac{h(h+20)}{2} \cdot 2$

solve for h - (set equal to zero and factor)

$1053 = h(h+20)$

$1053 = h^2 + 20h$

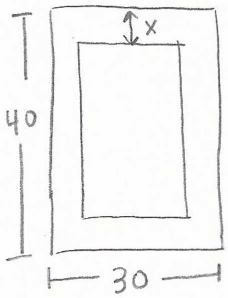
$-1053 \qquad -1053$

$0 = h^2 + 20h - 1053$

* This problem becomes prime. This is as far as you know at this point. on the test, you will be able factor.

11. You need more information. There will be enough information on the test.

12. Rectangular garden: 30 x 40 feet. uniform walkway around the garden.



new area: $(\frac{1}{2})(1200) = 600$

$600 = (30-2x)(40-2x)$

$600 = 1200 - 60x - 80x + 4x^2$

$0 = 4x^2 - 140x + 600$

$= 4(x^2 - 35x + 150)$

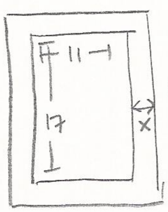
$= 4(x-5)(x-30)$

$x = 5 \quad \cancel{x = 30}$

This does not make sense in the context of the problem.

The walkway should be 5 feet wide.

13. writing portion: 11 x 17



$A = 315 in^2$

$315 = (11+2x)(17+2x)$

$315 = 187 + 22x + 34x + 4x^2$

$0 = 4x^2 + 56x - 128$

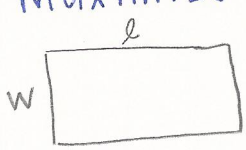
$0 = 4(x^2 + 14x - 32)$

$0 = 4(x-2)(x+16)$

$x = 2 \quad \cancel{x = -16}$

The border of the frame should be 2 inches wide.

14. Maximize the area.



$2w + 2l = 36$

$2l = -2w + 36$

$l = -w + 18$

$A = lw$

$A = w(-w+18)$

$A = -w^2 + 18w$ Find the vertex.

$A = -(w^2 - 18w + \_\_) - (-1)\_\_$

$A = -(w^2 - 18w + 81) - (-1)(81)$

$A = -(w-9)^2 + 81$

vertex: $(9, 81)$

$2(9) + 2l = 36$

$18 + 2l = 36$

$2l = 18$

$l = 9$

The dimensions of the pen should have a length of 9 feet and a width of 9 feet. The maximum area would be 81 feet².

15. 3 positive consecutive integers, sum of their squares is 149.

1st number = x

2nd number = x+1

3rd number = x+2

$(x)^2 + (x+1)^2 + (x+2)^2 = 149$

$x^2 + x^2 + 2x + 1 \quad x^2 + 4x + 4 = 149$

$3x^2 + 6x + 5 = 149$

$3x^2 + 6x - 144 = 0$

$3(x^2 + 2x - 48) = 0$

$3(x+8)(x-6) = 0$

$\cancel{x = -8} \quad x = 6$

↑ not positive

$\boxed{\text{The three numbers are 6, 7, and 8.}}$

16. $V_0 = 39.2$

$h_0 = 0$

$h(t) = -4.9t^2 + 39.2t$

$34.3 = -4.9t^2 + 39.2t$

$0 = -4.9t^2 + 39.2t - 34.3$

$0 = -4.9(t^2 - 8t + 7)$

$0 = -4.9(t-7)(t-1)$

$t = 7 \quad t = 1$

The object first reaches a height of 34.3 meters at 1 second and then again at 7 seconds. Therefore, the object is at or above 34.3 meters for 6 seconds.

17. vertex form gives us the vertex of a quadratic. We want a quadratic in vertex form to find a maximum/minimum value.

18. Intercept form gives us the x-intercepts or zeros of a quadratic. We want a quadratic in intercept form if we want the zeros or x-intercepts.

19. In standard form, we can easily get a y-intercept of the quadratic.

20. $y = x^2 - 4x + 5$

a. complete the square

$y = (x^2 - 4x + \_\_) + 5 - \_\_$

$y = (x^2 - 4x + 4) + 5 - 4$

$y = (x-2)^2 + 1$

b. axis of symmetry: $x = 2$

c. vertex: $(2, 1)$

d. Maximum value is 1 when $x = 2$.

e. $f(0) = 0^2 - 4(0) + 5 = 5 \quad (0, 5)$

f. $0 = (x-2)^2 + 1$

$-1 = (x-2)^2$

$\sqrt{-1} = x - 2$

* $\sqrt{-1}$ is not a real number. There are no x-intercepts.

g.